E. Cosenza (ed), *Eurocode 8 Perspectives from the Italian Standpoint Workshop*, 53-65, © 2009 Doppiavoce, Napoli, Italy

APPLICATION OF BAYESIAN TECHNIQUES TO MATERIAL STRENGTH EVALUATION AND CALIBRATION OF CONFIDENCE FACTORS

Giorgio Monti, Silvia Alessandri

Dept. of Structural Engineering and Geotechnic, Sapienza University of Rome, Rome, Italy,
giorgio.monti@uniroma1.it

ABSTRACT
A fundamental phase in the assessment of existing reinforced concrete buildings and in their strengthening design is the knowledge procedure. This is based on the collection of different kinds of information regarding: a) the structural system configuration, b) the materials strength, c) the reinforcing steel details, and d) the conditions of the structural elements.
The Italian Code (OPCM 3431, 03-05-05, Allegato 2) as well as the most advanced International Codes (FEMA 356, EC8 Part 3) specifies data collection procedures and ensuing Confidence Factors (CF) to apply to the mean values of the materials strength, based on the completeness and reliability of the information gathered (the so called Level of Knowledge).
Difference in the knowledge procedure about the single structural parameters and the actual possibility of propagation of information gathered on single members unlikely can be accounted for by a single CF. This paper proposes a method for evaluation of material strength and calibration of the relevant CF based on a bayesian procedure; the procedure takes into account the reliability of the results of tests executed by different methods and gives an indication on how to join such information.

KEYWORDS
Confidence Factors, reinforced concrete, strength, non destructive testing methods.

1 INTRODUCTION

Uncertainties on seismic performances of existing buildings are taken into account in the Italian Code OPCM 3431, 3-05-05, Allegato 2, as well as the most advanced International Codes (FEMA 356, Eurocode 8 Part 3) by different safety factors and analysis procedures than new buildings. The existing structures are characterized by uncertainties of intrinsic and epistemic kind; they mainly depend on completeness of information on geometrical and mechanical elements characteristics and also on the possibility they could contain hidden gross errors and may have been submitted to previous earthquake or other accidental actions with unknown effects. The codes specify data collection procedures about the configuration of the structural system, as well as material strength and condition of the structural elements comprising the building. This data shall be obtained from available drawings, specifications, and other documents for the existing construction, and shall be supplemented and verified by on-site investigations, including destructive and non-destructive examination and testing of building materials and components. As a function of the completeness of as-built information

on buildings (Level of Knowledge) the Codes specify different Confidence Factors (CF) to be applied to mean strength values of materials.
OPCM 3431 and EC8-Part 3 follow the same procedure; they define three knowledge levels:

- KL1 : Limited knowledge
- KL2 : Normal knowledge
- KL3 : Full knowledge

The factors determining the knowledge level are:

- geometry: geometrical properties of the structural system and of nonstructural elements that may affect structural response;
- details: amount and detailing of reinforcement in reinforced concrete, connections between steel members, connection of floor diaphragms to lateral resisting structure, bond and mortar jointing of masonry and nature of any reinforcing elements in masonry;
- materials: mechanical properties of the constituent materials.

The input data shall be collected from a variety of sources, including:

- available documentation specific to the building in question,
- relevant generic data sources (e.g. contemporary codes and standards),
- field investigations and in-situ and/or laboratory measurements and tests.

The classification of the levels of inspection and testing depends on the percentage of structural elements that have to be checked for details, as well as on the number of material samples per floor that have to be taken for testing. For ordinary situations recommended minimum values are given. Mean value properties of the existing materials obtained from *in-situ* tests and from the additional sources of information must be scaled by the Confidence Factor, accounting for the level of knowledge attained and, implicitly, the reliability of all the information collected on the building.
Difference in the knowledge procedure about the single structural parameters and the actual possibility of propagation to the structure as a whole of information gathered on single members unlikely can be accounted for by a single CF to be applied to mean materials strength values. Material strength is characterized by an intrinsic spatial variability and an epistemic uncertainty caused by workmanship (for instance not compliance with the original project, execution of structural elements in different times with different materials strength), reliability of testing methods and degradation of material properties with the time. Otherwise amount and detailing of reinforcement, defective detailing, etc., neglecting the intrinsic uncertainties, are characterized by epistemic uncertainties only, mainly due to lack of the original project and/or not compliance with it; collected data on one structural element are certain but don’t allow to erase uncertainties about other elements. Therefore, owing to the different nature of the knowledge process and data type for a reinforced concrete structure, it should be better to distinguish between information obtained on materials strength and information obtained on amount and detailing of reinforcement and to define two different procedures: a) a procedure for data processing and evaluation of a Confidence Factor for materials strength; b) a procedure for the assessment of reinforcing details.
Another aspect of the problem is that the use of different testing methods (destructive and nondestructive) on the same concrete gives information with different reliability. Therefore the matter is how to joint such information, taking into account that the reliability of concrete strength given by non-destructive testing is greatly influenced by reliability of the used regression curves. These haven’t got a general validity and should be calibrated every time.

The Codes define the minimum number of destructive and non destructive tests that can be executed on a single building but don't take explicitly into account the reliability of each testing method and don't specify how to joint test results.

In this paper a procedure for evaluation of material strength and calibration of the relevant CF is developed, based on the application of the bayesian method. The bayesian method allows to take into account the reliability of the information collected on the material strength; destructive and non-destructive testing results are separately employed, taking into account the reliability of each testing method, to up-date a prior probability distribution function. By the developed method a reference parameter for materials strength is evaluated as the lower bound of a confidence interval for the Bayesian mean. A correlation equation is calibrated to evaluate the CF as a function of the number, the kind and the reliability of each testing employed and of the reliability of prior information, so that it can be applied to a design value of material strength parameter to make it equal to the reference parameter. The design value is a weighting mean of strength values obtained by testing and by prior information.

1.1 Prior knowledge

Prior knowledge of material strength is based on construction documents, reports, reference standards and codes from the period of construction. From this data a mean value of the material strength, μ'_f, and the relating variance, $\sigma'^2_{\mu_f}$ are evaluated.

1.2 Posterior knowledge

The material strength, f, can be statistically described by a lognormal distribution function, that is usually used to describe the probabilistic model of the concrete material properties.

If the variable f is lognormal its natural logarithm, $x = \ln(f)$, is a normal random variable with mean value λ and standard deviation ζ.

The posterior distribution, $f_\Lambda(\lambda)$, of λ is normal with the following statistics:

$$\mu''_\lambda = \frac{\mu'_\lambda \left(\zeta^2_{\lambda,p} / n_p\right) + \bar{x}_{\lambda,p}\sigma'^2_\lambda}{\left(\zeta^2_{\lambda,p} / n_p\right) + \sigma'^2_\lambda} \tag{1}$$

$$\sigma''^2_\lambda = \frac{\left(\zeta^2_{\lambda,p} / n_p\right) \cdot \sigma'^2_\lambda}{\left(\zeta^2_{\lambda,p} / n_p\right) + \sigma'^2_\lambda} \tag{2}$$

where: μ'_λ and σ'^2_λ are the prior mean value and variance, respectively; $\bar{x}_{\lambda,p}$ and $\zeta'^2_{\lambda,p}$ are the mean value and variance of the natural logarithm of test results, respectively.

More than one test method can be employed performing consecutive up-dating of the probability distribution function. Destructive and non-destructive testing results are separately employed, taking into account individual testing reliability.

If n_{DM} is the destructive tests number performed and $f_{i,DM}$ is the strength value from the i-*th* test, the mean sampling value of destructive testing is:

$$\overline{x}_{DM} = \frac{\sum_{i=1}^{n_{DM}} \ln(f_{c_{i,DM}})}{n_{DM}} \tag{3}$$

The evaluation of the sampling variance ζ^2_{DM} can take into account testing errors and errors in regression curve that provides the material resistance as a function of the testing parameter:

$$\zeta^2_{DM} = \zeta^2_{s,DM} + \zeta^2_{t,DM} \tag{4}$$

where $\zeta^2_{s,DM}$ the variance of the natural logarithm of data, given by:

$$\zeta^2_{s,DM} = \frac{\sum_{i=1}^{n_{DM}} \left[\ln(f_{c_{i,DM}}) - \overline{x}_{DM} \right]^2}{n_{DM} - 1} \tag{5}$$

and $\zeta^2_{t,DM}$ is the variance of the regression curve, given by:

$$\zeta^2_{t,DM} = CoV_{t,DM} \cdot \overline{x}_{DM} \tag{6}$$

where $CoV_{t,DM}$ is the coefficient of variation of the regression curve.
When the mean value and variance are known the first updating of the statistics of the distribution, $f_\Lambda(\lambda)$ of λ is possible:

$$\mu''_\lambda = \frac{\mu'_\lambda \left(\zeta^2_{DM} / n_{DM} \right) + \overline{x}_{DM} \sigma'^2_\lambda}{\left(\zeta^2_{DM} / n_{DM} \right) + \sigma'^2_\lambda} \tag{7}$$

$$\sigma''^2_\lambda = \frac{\left(\zeta^2_{DM} / n_{DM} \right) \cdot \sigma'^2_\lambda}{\left(\zeta^2_{DM} / n_{DM} \right) + \sigma'^2_\lambda} \tag{8}$$

A second updating is possible by using non destructive testing results.
If n_{NDM} is the non-destructive tests number performed and $f_{i,NDM}$ is the strength value from the i-*th* test, the mean sampling value of destructive testing is:

$$\overline{x}_{NDM} = \frac{\sum_{i=1}^{n_{NDM}} \ln(f_{c_{i,NDM}})}{n_{NDM}} \tag{9}$$

The evaluation of the sampling variance ζ^2_{NDM} can take into account testing errors and errors in the regression curve that provides the material resistance as a function of the testing parameter:

$$\zeta^2_{NDM} = \zeta^2_{s,NDM} + \zeta^2_{t,NDM} \tag{10}$$

where $\zeta^2_{s,NDM}$ the variance of the natural logarithm of data, given by:

$$\zeta^2_{s,NDM} = \frac{\sum_{i=1}^{n_{NDM}} \left[\ln(f_{c_{i,NDM}}) - \bar{x}_{NDM}\right]^2}{n_{NDM} - 1} \quad (11)$$

and $\zeta^2_{t,NDM}$ is the variance of the regression curve, given as:

$$\zeta^2_{t,NDM} = CoV_{t,NDM} \cdot \bar{x}_{NDM} \quad (12)$$

where $CoV_{t,NDM}$ is the coefficient of variation of the regression curve.
A second updating of the statistics of the distribution, $f_\Lambda(\lambda)$ of λ is now possible:

$$\mu'''_\lambda = \frac{\mu''_\lambda \left(\zeta^2_{NDM} / n_{NDM}\right) + \bar{x}_{NDM} \sigma''^2_\lambda}{\left(\zeta^2_{NDM} / n_{NDM}\right) + \sigma''^2_\lambda} \quad (13)$$

$$\sigma'''^2_\lambda = \frac{\left(\zeta^2_{NDM} / n_{NDM}\right) \cdot \sigma''^2_\lambda}{\left(\zeta^2_{NDM} / n_{NDM}\right) + \sigma''^2_\lambda} \quad (14)$$

Replacing the value of μ''_λ and σ''^2_λ in the previous equation the posterior statistics value are sought:

$$\mu'''_\lambda = \frac{\frac{\mu'_\lambda}{(\sigma'_\lambda)^2} + \frac{n_{DM} \cdot \bar{x}_{DM}}{(\zeta_{DM})^2} + \frac{n_{NDM} \cdot \bar{x}_{NDM}}{(\zeta_{NDM})^2}}{\frac{n_{DM}}{(\zeta_{DM})^2} + \frac{n_{NDM}}{(\zeta_{NDM})^2} + \frac{1}{(\sigma'_\lambda)^2}} \quad (15)$$

$$\sigma'''^2_\lambda = \frac{1}{\sqrt{\frac{n_{DM}}{(\zeta_{DM})^2} + \frac{n_{NDM}}{(\zeta_{NDM})^2} + \frac{1}{(\sigma'_\lambda)^2}}} \quad (16)$$

1.3 Confidence interval on mean value

It's possible to improve mean value statistics reliability by applying the confidence interval for the mean. What is of interest is the lower confidence limit $<\mu)_{1-\alpha}$, which is the value that the population mean will be larger with a confidence level of $(1-\alpha)$. For a Normal distribution function with unknown variance it is given by:

$$P\left(\frac{\bar{x} - \mu}{s/\sqrt{n}} \le t_{\alpha/2,n-1}\right) = 1 - \alpha \quad (17)$$

where: $\bar{x}$ and s are the sampling mean and standard deviation, respectively; n is the sampling dimension; μ is the mean of the population from which the sample is observed; $(1-\alpha)$ is the specified level of confidence and $t_{\alpha/2,n-1}$ is the value of the t-Student variate with $n-1$ degrees of freedom having a cumulative probability level $\alpha/2$. Rearranging the Eq. (17) the lower limit of the confidence interval $(1-\alpha)$ is obtained:

$$\bar{x} \geq \mu - t_{\alpha/2,n-1} \cdot s/\sqrt{n} \tag{18}$$

A 95% lower confidence level is here considered:

$$\mu'''_{\lambda,\inf} = \mu'''_{\lambda} - t_{\alpha/2,n-1}\sigma'''_{\lambda} \tag{19}$$

Introducing the expression of μ'''_{λ} in the Eq. (19) we get:

$$\mu'''_{\lambda,\inf} = \sigma'''_{\lambda}\left\{(\sigma'''_{\lambda})\left[\frac{\mu'_{\lambda}}{(\sigma'_{\lambda})^2} + \frac{n_{MD}\cdot\bar{x}_{MD}}{(\zeta_{MD})^2} + \frac{n_{MND}\cdot\bar{x}_{MND}}{(\zeta_{MND})^2}\right] - t_{\alpha/2,n-1}\right\} \tag{20}$$

The parameter $\mu'''_{\lambda,\inf}$ is related to the variable $\lambda = E[\ln(f)]$; from it it's possible to evaluate the parameter $\breve{m}'''_{\inf,\breve{m}_f}$, the lower confidence limit for the Bayesian median value, $\breve{m}'''_{\breve{m}_f}$, of median value, $\breve{m}_f$, of the material strength, f, that is the value with a 50 % probability:

$$\breve{m}'''_{\inf,\breve{m}_f} = e^{\mu'''_{\lambda,\inf}} \tag{21}$$

1.4 Definition of the design material strength

The parameter $\breve{m}'''_{\inf,\breve{m}_f}$ represents the value for the structural assessment; in order to facilitate its evaluation a simplified procedure is defined. The value $\breve{m}'''_{\inf,\breve{m}_f}$ can be obtained by scaling with an opportune Confidence Factor a weighted mean, μ, of the sampling mean vales obtained by the different testing methods and the prior information:

$$\mu_D = \frac{\mu}{FC} \cong \breve{m}'''_{\inf,\breve{m}_f} \tag{22}$$

where:

$$\mu = \left[\frac{\mu'_f + n_{DM}\cdot\bar{x}_{DM} + n_{NDM}\cdot\bar{x}_{NDM}}{1 + n_{DM} + n_{NDM}}\right] \tag{23}$$

where $\bar{x}_{DM}$ and $\bar{x}_{NDM}$ are the sampling mean of the destructive and non destructive tests, respectively; n_{DM} and n_{NDM} are the corresponding sampling dimension. Generally, if M_i is the *i*-th testing method adopted, the material strength for the assessment is:

$$\mu = \left[\frac{\mu'_f + \sum_i n_{M_i} \cdot \bar{x}_{M_i}}{1 + \sum_i n_{M_i}} \right] \tag{24}$$

where $\bar{x}_{Mi}$ is the sampling mean of the i-th testing method and n_{Mi} its dimension.

1.5 Calibration of Confidence Factors for concrete strength

The CF can be expressed in an explicit form as a function of the Bayesian coefficient of variation V_μ for the median value of the material strength:

$$FC = a + c \cdot V_\mu^\omega \tag{25}$$

The parameter V_μ, which estimate the reliability of available information, is defined as:

$$V_\mu = \frac{\sigma_\mu}{\mu_\mu} = \frac{\sqrt{\sum_i \frac{n_{Mi}}{s_{s,Mi}^2 + s_{t,Mi}^2}}}{\sum_i \frac{n_{Mi} \cdot \bar{x}_{Mi}}{s_{s,Mi}^2 + s_{t,Mi}^2}} \tag{26}$$

where $s_{s,Mi}^2$ and $s_{t,Mi}^2$ are the sampling variance and the variance of the regression curve of the *i*-th testing method, respectively. The Eq. (25) has been calibrated for concrete strength by applying the least squares method. A Monte Carlo method has been used to simulate sampling with destructive and non destructive testing. The simulated samplings are extracted from a population with median concrete strength $\breve{m}_{fc}$ ranging from 10 MPa to 40 MPa and hypothesizing the possible range of all the parameters ($V_{fc}, \mu'_{fc}, V'_{fc}, \mu'_{fc}, n_{DM}, V_{t,DM}, n_{NDM}, V_{t,NDM}$). The parameters a, c and ω have been evaluated by applying the least squares method to the set of values μ_D calculated by Eq. (22) with FC given by (25), and the set of $\breve{m}'''_{\inf,\breve{m}_{fc}}$ calculated by the Bayesian procedure described above, so that the condition given in Eq. (22) is met. The resulting equation for the CF is the following:

$$FC = 0.9 + \sqrt{V_\mu} \tag{27}$$

The equation (27) is effective if samples have been extracted from homogeneous zones of the structure. If in the structure potential non homogeneous zones are identified, the t-Student test can be executed on the mean vales extracted from the two zones.
For two independent samples, with homogeneous variance, the t-Student test is executed by verifying the following condition:

$$\frac{\left(\bar{X}_A - \bar{X}_B\right) - \left(\mu_A - \mu_B\right)}{\sqrt{S_p^2 \cdot \left(\frac{1}{n_A} + \frac{1}{n_B}\right)}} \le t_{\alpha/2,(n_A+n_B-2)} \tag{28}$$

where:

- $\bar{X}_A$ and $\bar{X}_B$ are the sampling means of the sample extracted from A and B zones, respectively;
- μ_A and μ_B are the expected mean values; if A and B are homogeneous zone the following condition is met: $\mu_A - \mu_B = 0$;
- n_A and n_B are the sampling dimensions of A and B;
- S_p^2 is the pooled variance given by:

$$S_p^2 = \frac{(n_A - 1)s_A^2 + (n_A - 1)s_B^2}{n_A + n_B - 2} \tag{29}$$

- $t_{\alpha/2,(n_A+n_B-2)}$ is the value of the t-Student variate with $n_A + n_B - 2$ degrees of freedom having a cumulative probability level $\alpha/2$.

If the t-student test identify non homogeneous zones these must be separately evaluated considering two different median value for concrete strength with two CF.

1.6 Summary of the Procedure

Procedure for evaluation of CF for material strength is based on the following steps:

- acquisition of prior information;
- determination of possible non homogeneous zones;
- choice of destructive and non destructive testing methods to be applied;
- definition of coefficient of variation for each testing method in relation to the used regression curves;
- execution of destructive test in each homogeneous zone and evaluation of the mean value $\bar{x}_{DM}$ and variance $s_{s,DM}^2$ for each zone;
- execution of non destructive tests in each homogeneous zone and evaluation of the mean values $\bar{x}_{NDM,i}$ and variance $s_{s,NDM,i}^2$ for each zone;
- evaluation of V_μ by equation (26);
- evaluation of FC by equation (27).

1.7 Evaluation of the correlation equations between concrete strength and non-destructive testing parameters and determination of CoV for material strength

Functional relations that give the concrete strength value from non-destructive testing results are defined by regression analysis on data from destructive testing results (cores).

Concrete strength, f_c, is evaluated as a function f of testing parameters $\mathbf{X} = (x_1, \ldots, x_k)$:

$$f_c = f(\mathbf{X}, \boldsymbol{\theta}) + \varepsilon \tag{30}$$

where $\boldsymbol{\theta}$ is the parameter vector and ε is a r.v., with unit mean value and standard deviation σ, taking into account errors in the definition of the functional relation f. The function f is usually defined as a polynomial in $\mathbf{X}$, with $\boldsymbol{\theta}$ coefficient vector ($\boldsymbol{\theta} = (\beta_1, \ldots, \beta_k, \sigma^2)$) ; the probability distribution function of f is studied in the context of a set experiments $i = 1, \ldots n$

on which f_c and $\mathbf{X}$ are measured. The vector f_c is the vector of outcomes of destructive testing (cores) by which the correlation equations of non-destructive testing methods (rebound, ultrasonic pulse velocity, Sonreb, etc.) are calibrated; the f_c vector has n elements, corresponding to the n test outcomes (observations). $\mathbf{X}$ is a $n \times k$ matrix, with $k =$ predictors number, corresponding to the testing methods included in the correlation equation. If the regression includes an intercept, one of the columns of X is a column of ones. The parameters in $\boldsymbol{\theta}$ are the regression coefficients β_i and the error variance of the fitted model, σ^2. The $i-th$ element in f_c is given by:

$$f_{ci} = \beta_1 x_{i1} + \ldots + \beta_k x_{ik} + \varepsilon \tag{31}$$

The probability distribution function of the r.v ε is assumed to be Normal; under the hypothesis of independent errors and with equal variance, the probability distribution function of f_c given parameters β and σ^2 and predictors $\mathbf{X}$ is a normal distribution with mean $\mathbf{X}\beta$ and variance σ^2:

$$\left(f_c \mid \beta, \sigma^2, \mathbf{X}\right) \sim N\left(\mathbf{X}\boldsymbol{\beta}, \mathbf{I}\sigma^2\right) \tag{32}$$

$$P\left(f_c \mid \beta, \sigma^2, \mathbf{X}\right) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\frac{\left(f_c - \mathbf{X}\boldsymbol{\theta}\right)^2}{\sigma^2}\right] \tag{33}$$

where $\mathbf{I}$ is the identity matrix $n \times n$.

Coefficients in $\boldsymbol{\theta}$ are generally unknown and can be estimated by a regression analysis on the in-situ test results; advantages in application of the bayesian inference method lies mainly in deriving conclusion on the parameters $\boldsymbol{\theta}$ and on the data in a probability statement.

The method presented below come from the Bayesian theory for normally-distributed random variables (Gelman et al. 1995).

By the bayesian inference, once the regression model has been defined the probability distribution of parameters conditional on the data, $p\left(\boldsymbol{\theta} \mid f_c\right)$, and the predicted distribution of unobserved data, $p\left(\tilde{f}_c \mid f_c\right)$, can be evaluated. By applying the Bayes' rule, the posterior distribution function, $p\left(\boldsymbol{\theta} \mid f_c\right)$, is given by:

$$p\left(\boldsymbol{\theta} \mid f_c\right) = \frac{p\left(\boldsymbol{\theta}, f_c\right)}{p\left(f_c\right)} = \frac{p\left(\boldsymbol{\theta}\right) p\left(f_c \mid \boldsymbol{\theta}\right)}{p\left(f_c\right)} \tag{34}$$

where $p\left(\boldsymbol{\theta}\right)$ is the prior distribution function of the parameters, $p\left(f_c \mid \boldsymbol{\theta}\right)$ is the sample distribution and $p\left(f_c\right) = \sum_{\theta} p\left(\boldsymbol{\theta}\right) p\left(f_c \mid \boldsymbol{\theta}\right)$. An alternative form of the Eq. (34) omits the term $p\left(f_c\right)$ which doesn't depend on $\boldsymbol{\theta}$ and can be considered constant:

$$p\left(\boldsymbol{\theta} \mid f_c\right) \propto p\left(\boldsymbol{\theta}\right) p\left(f_c \mid \boldsymbol{\theta}\right) \tag{35}$$

Alternatively the joint posterior distribution of β and σ^2 can be expressed as the conditional probability of β given σ^2 times the marginal posterior probability of σ^2:

$$p\left(\beta,\sigma^2 \mid f_c\right) = p\left(\beta \mid \sigma^2, f_c\right) p\left(\sigma^2 \mid f_c\right) \tag{36}$$

Under the assumption of Normal errors, independent and with equal variance, the coefficients estimates are also normally distributed:

$$\beta \mid \sigma^2, f_c \sim N\left(\hat{\boldsymbol{\beta}}, \mathbf{V}_\beta \sigma^2\right) \tag{37}$$

where:

$$\hat{\boldsymbol{\beta}} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{f}_c \tag{38}$$

$$\mathbf{V}_\beta = \left(\mathbf{X}^T\mathbf{X}\right)^{-1} \tag{39}$$

The marginal posterior distribution of σ^2 is an $Inv-\chi^2$ with $\nu = n-k$ degree of freedom:

$$\sigma^2 \mid f_c \sim Inv-\chi^2\left(n-k, s^2\right) \tag{40}$$

where:

$$s^2 = \frac{1}{n-k}\left(\mathbf{f}_c - \mathbf{X}\hat{\boldsymbol{\beta}}\right)^T\left(\mathbf{f}_c - \mathbf{X}\hat{\boldsymbol{\beta}}\right) \tag{41}$$

The posterior marginal distribution of $\beta \mid f_c$ is a multivariate t-Student with $n-k$ degree of freedom:

$$p\left(\beta \mid f_c\right) = \int p\left(\beta \mid \sigma^2, f_c\right) p\left(\sigma^2 \mid f_c\right) d\sigma^2 \tag{42}$$

The predictive conditional posterior distribution for new data $\tilde{f}_c$, given σ^2, is also Normal with mean value:

$$\begin{aligned} E\left(\tilde{f}_c \mid \sigma^2, f_c\right) &= E\left(E\left(\tilde{f}_c \mid \beta, \sigma^2, f_c\right) \mid \sigma^2, f_c\right) \\ &= E\left(\tilde{X}\beta \mid \sigma^2, f_c\right) \\ &= \tilde{\mathbf{X}}\tilde{\boldsymbol{\beta}} \end{aligned} \tag{43}$$

and variance:

$$var\left(\tilde{f}_c \mid \sigma^2, f_c\right) = \left(\mathbf{I} + \tilde{\mathbf{X}}\mathbf{V}_\beta\tilde{\mathbf{X}}^T\right)\sigma^2 \tag{44}$$

The marginal posterior distribution for new observations, $\tilde{f}_c$, $p\left(\tilde{f}_c \mid f_c\right)$, averaging over σ^2, is a multivariate t-Student with mean value $\tilde{\mathbf{X}}\tilde{\boldsymbol{\beta}}$, squared scale matrix $\left(\mathbf{I} + \tilde{\mathbf{X}}\mathbf{V}_\beta\tilde{\mathbf{X}}^T\right)s^2$ and $\nu = n - k$ degree of freedom:

$$p\left(\tilde{f}_c \mid f_c\right) = \text{multivariate } t\left[n-k, \tilde{\mathbf{X}}\tilde{\boldsymbol{\beta}}, \left(\mathbf{I} + \tilde{\mathbf{X}}\mathbf{V}_\beta\tilde{\mathbf{X}}^T\right)s^2\right] \tag{45}$$

Therefore the variance of $\tilde{f}_c$ is given by:

$$var\left(\tilde{f}_c \mid f_c\right) = \frac{\nu}{\nu - 2}\left(\mathbf{I} + \tilde{\mathbf{X}}\mathbf{V}_\beta\tilde{\mathbf{X}}^T\right)s^2 \tag{46}$$

1.8 Application to Existing Buildings

The proposed method has been applied to evaluation of median concrete strength for an existing building.

Destructive (cores) and non destructive (sclerometer and ultrasonic pulse velocity) testing has been executed. No prior information was available. Non destructive testing results have been combined by the Sonreb method, using a regression curve calibrated on data from destructive tests results, as previously illustrated, and two regression curves taken from literature with relevant coefficients of variation:

$$f_c = 7.695 \cdot 10^{-10} \cdot I^{1.4} \cdot V^{2.6} \quad (Kg/cm^2; m/s) \qquad \text{RILEM 43 CND} \tag{47}$$

$$f_c = 1.2 \cdot 10^{-9} \cdot I^{1.058} \cdot V^{2.446} \quad (MPa; m/s) \qquad \text{Di Leo et al.} \tag{48}$$

where I is the rebound number and V is the ultrasonic pulse velocity.

For the Sonreb regression curve evaluated on tests results, the mean value and the standard deviation have been calculated as in § 1.7; for the regression curves given by Eq. (47) and Eq. (48), under the hypothesis that the two parameters I and V are independent, the mean value and the variance of concrete strength are evaluated by the following equations:

$$\bar{x}_{NDM} = f\left(\bar{I}, \bar{V}\right) \tag{49}$$

$$s_{NDM}^2 = \left(\frac{\partial f_c}{\partial I}\right)_{\bar{I}}^2 \cdot s_I^2 + \left(\frac{\partial f_c}{\partial V}\right)_{\bar{V}}^2 \cdot s_V^2 + V_{NDM}^2 \cdot \bar{x}_{NDM}^2 \tag{50}$$

where $\bar{I}$ and $\bar{V}$ are the mean value of the rebound number and of the ultrasonic pulse velocity, respectively; $\bar{x}_{MND}$ is the mean value of concrete strength obtained from application of Sonreb method, $f\left(\bar{x}_{MND}\right)$ is expression of the regression curve; s_I^2 and s_V^2 are the variance of the non-destructive parameters; finally, V_{NDM} is the coefficients of variation of Sonreb curves, taken form Giannini e al. (2003).

Figure 1 and Figure 2 show the CF variation compared to the variation of mean values of concrete strength obtained by destructive and nondestructive methods and to the relevant coefficient of variation; they point out that the CF correctly reflect the information reliability.

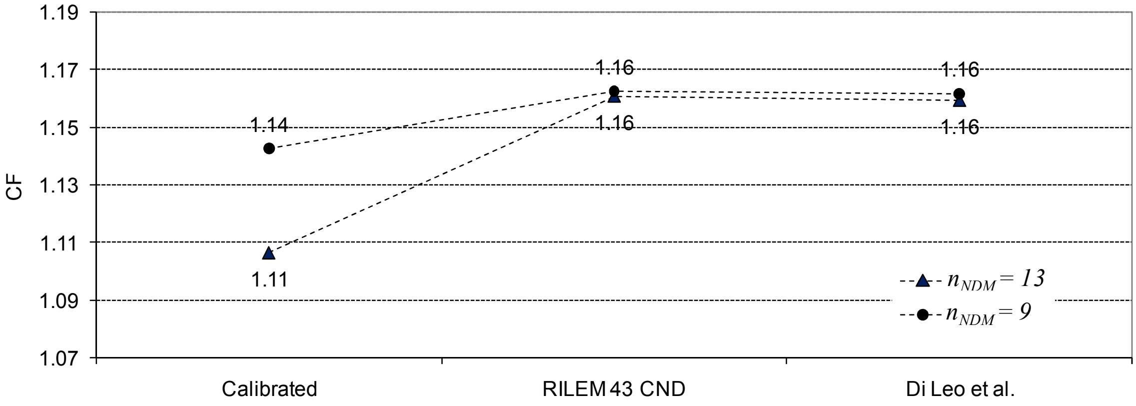

Figure 1. CF vs n_{NDM} for different Sonreb equations.

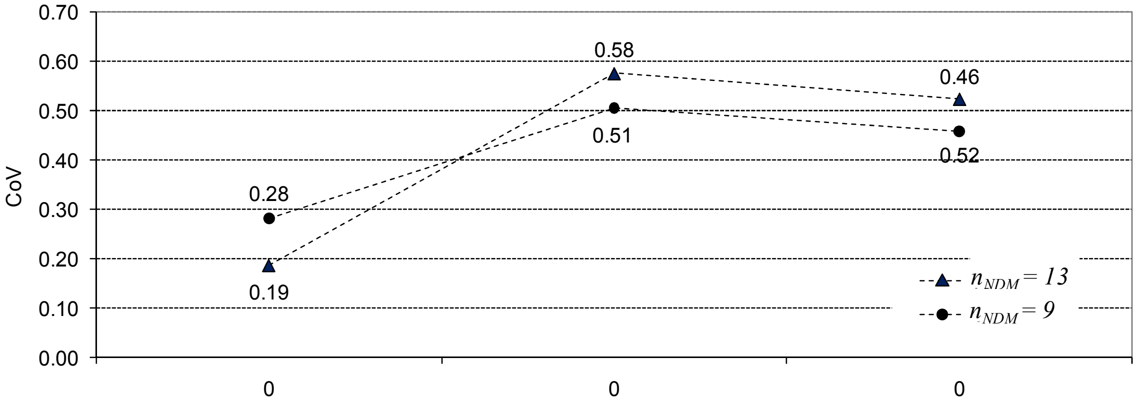

Figure 2. CoV vs n_{NDM} for different Sonreb equations.

2 CONCLUSIONS

This paper proposes a procedure for evaluation of CF which goes beyond the definition given by Italian (OPCM 3431) and European (EC8-Part3) Codes. This is because of the different nature of the knowledge process and uncertainties which characterize material strength and reinforcement detailing. The paper proposes the definition of different CF and an expression to evaluate CF for material strength as a function of scattering of testing data and prior information. The proposed procedure has been calibrated on simulated cases and tested by an existing building material strength evaluation. The application points out that the CF correctly reflects the informations reliability.

3 ACKNOWLEDGEMENTS

This work has been carried out under the program “Dipartimento di Protezione Civile-Consorzio RELUIS”, signed on 2005-07-11 (n. 540), Research Line 2, whose financial support was greatly appreciated.

4 REFERENCES

Di Leo A. et al. “Prove non distruttive sulle costruzioni in cemento armato”, *Sistema di Qualità e prove non Distruttive per l’affidabilità e la sicurezza delle strutture civili.*

EC8 Eurocodice 8 – Parte 3 ENV 1998 1-1. Design of structures for earthquake resistance. 2004.

Gelman, A., Carlin, J.B., Rubin, D.B. (1995). “Bayesian Data Analysis”. Chapman & Hall, 1995.

Ordinanza del Presidente del Consiglio dei Ministri (OPCM) n. 3431, Ulteriori modifiche ed integrazioni all'ordinanza del Presidente del Consiglio dei Ministri n. 3274 del 20 marzo 2003. Gazzetta Ufficiale della Repubblica Italiana n. 107 del 10-5-2005 (Suppl. Ordinario n.85), 2005.

Ordinanza del Presidente del Consiglio dei Ministri (OPCM) n. 3519. Criteri per l’individuazione delle zone sismiche e la formazione e l’aggiornamento degli elenchi delle medesime zone. Gazzetta Ufficiale della Repubblica Italiana n. 108, 2006.

Giannini R., Sguerri L. (2004). “Tecniche bayesiane per la stima delle leggi di correlazione tra velocità ultrasonica e resistenza del calcestruzzo”, *Giornate AICAP 2004*, Verona 2004, pp. 553-560.

Giannini R., Sguerri L., Ninni V. (2003), “Affidabilità dei metodi d’indagine non distruttivi per la valutazione della resistenza del calcestruzzo”, *10° Congresso Nazionale dell’AIPND*, Ravenna, pp. 670-679.

Monti G., Alessandri S. (2008). “Confidence factors for concrete and steel strength”, *Convegno RELUIS “Valutazione e riduzione della vulnerabilità sismica di edifici esistenti in cemento armato”*, Roma 29-30 maggio 2008.

Monti, G., Alessandri S., Goretti, A., (2007). “Livelli di conoscenza e fattori di confidenza”. *Proceedings XII Convegno ANIDIS*, Giugno 2007.

Sguerri L., Paolacci F., Defelice G., Giannini R. (2006). “Stima della resistenza del calcestruzzo delle capriate in cemento armato dell’ex mattatoio di Roma mediante misure non distruttive”; *Convegno Nazionale Sperimentazione su Materiali e Strutture*, Venezia 6-7 Dicembre 2006.