ELASTIC PERIOD OF EXISTING RC-MRF BUILDINGS

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ABSTRACT
The fundamental period structures has a primary role in the seismic design and assessment and design as it is the main feature of the structure allowing to determine, at least, the elastic demand and is the basis to assess the required inelastic performance in static procedures. In fact, the definition of easy to manage relationships for the assessment of the elastic period has been the subject of a significant deal of research of both experimental and numerical/analytical studies, some of which has been acknowledged by codes and guidelines worldwide. Moreover, this kind of information is useful for territorial-scale seismic loss assessment methodologies. In the most of the cases the assessment of the period is considered as function of the structural system classification and number of storeys. Reinforced concrete (RC) buildings constituting the most of the building stock in Italy and in seismic prone areas in Europe, were built after the second world war and are designed with obsolete seismic codes if not for gravity loads only. Therefore, a significant variability of the structural system may affect a class of buildings featuring the same height and/or number of storeys. This, along the contribution of the stair module, may affect the elastic periods in the two main directions of a three-dimensional building. In the study presented these issues are investigated with reference to a population of existing RC structures designed via the practice at the time of supposed construction (e.g., simulated design) and with reference to relative enforced code. Elastic period is evaluated for both main directions of the buildings of the considered sample, and regression analysis is employed to capture the dependency of the elastic dynamic properties of the structures as a function of mass and stiffness.

KEYWORDS
Elastic period, sub-standard, RC buildings, structural system.

1 INTRODUCTION
In static procedures for seismic structural assessment the fundamental period is one of global characteristics to determine effects of seismic action in terms of horizontal forces. In the dynamic procedures (e.g., nonlinear) it is necessary to select the appropriate hazard information and input ground motions, especially for first-mode dominated structures. Generally speaking, the period relates seismic demand to capacity allowing to determine the seismic performance and therefore the safety level. The most of the seismic codes worldwide propose to easy to apply relationships to determine the elastic period as a function of height or number of storeys given the structural typology (SEAOC, 1998, Eurocode 8, 2004). Such relationships, especially those for moment-resisting RC frames, have been calibrated on experimental studies, which have become a standard
reference at an international level (ATC, 1978, Goel and Chopra, 1997). These important studies are based on seismic monitoring of buildings subjected to seismic actions, eventually repeated because of more earthquakes hit the structure, in significantly seismic prone areas in which earthquake resistant design is well established since long time. Recently, research effort was devoted to the attempt of develop similar relationships, for European “typical” frames and not buildings, for the so-called effective- or yield- period which is that of interest for determining the non-linear demand in those cases where the capacity derives from static push-over analysis (Crowley and Pinho, 2004). This period, which is longer than the elastic has more to do with the yielding and/or cracked stiffness of the structure and is more easily assessed via analytical/numerical procedures, although results strictly depend on the structural modeling assumptions.

It is well known as the existing RC buildings in Italy, significant portion of the building stock, have been designed and erected mainly after the second world (e.g., in the 1940-1980 period) war when only a fraction of the territory was considered as seismically prone. Design was carried out by for gravity loads only, and also when consideration of seismic action was required, it resulted in the application of period-independent horizontal forces without regard to capacity design which is the fundamental of earthquake resistant design nowadays. A class of gravity-load designed buildings may feature a structural system which may be heterogeneous as the plan distribution of resisting frames may not follow the regularity principle established in the seismic case. This is also useful to point out that may be not appropriate, when analyzing these building to refer to frames as the two main directions may have dissimilar dynamic properties reflecting the variability of the structural system. Moreover, the stair-module cannot also be neglected when investigating this topic. Similarly, the “seismic” buildings of the time-span given, although presenting a more rational structural system in respect to seismic actions, are expected to not show the stiffness and regularity features of a modern earthquake resistant building.

The study presented in this paper investigated these issues for classes of existing RC buildings and how they reflect on the elastic properties. In particular, the variability of the elastic period in the two directions is assessed analytically in respect to the variability of some parameters of characterizing the structural configuration. To this aim rectangular buildings with number of storeys between 2 and 8 have been considered, which are very common in Italy. The building are bare-frames, according to the codes, for which the stair is also considered. As the study refer to a numerical analysis of a class or population of buildings those had to be specifically designed. The simulated design was carried out via an automatic procedure (Verderame et al., 2008) which implements the design rules and professional practice at the time when the building are supposed to date. In particular, in the study refers to the Italian design principles, which represent the European and Mediterranean practice (Carvalho et al., 1999; Bal et al., 2007, Verderame et al., 2009).

Two different populations of buildings have been considered: (i) gravity-loads only; and (ii) sub-standard seismic design accounting for seismic action via statically-equivalent horizontal forces. From these two populations have been split in 14 classes of buildings with fixed number of storeys. The periods in the two main directions have been regressed versus the height, as suggested by codes and existing literature on the topic, to compare and to see how much of the variability is captured by the independent variable. Subsequently, other covariates which may explain the variability of mass and stiffness, related to the global dimensions of the building, have been included to better assess the influence on the elastic period of the design practice and structural peculiarities.

In the following, after a state-of-the-art review, of simplified relationships for elastic period estimation for RC buildings, the analytical procedure considered is described along with the
population of building considered as a sample for the analyses. Finally, results are presented and discussed with the aim of assessing the elastic periods and explaining its variability in respect to existing and code approaches.

2 RESEARCH BACKGROUND

Period is depending on those factors directly affecting structural mass and stiffness. Globally, proxies for the mass may be building’s global dimensions (e.g., plan area and number of floors), stiffness may be related with structural features and height. The most of the relationships to estimate the period are a function of global height (H) as it is a simple parameter, known before detailed design, which may explain the ratio between stiffness and mass of the building. Formulation of period-height relationships is typically of the type in Eq. (1) where \( \alpha \) is depending on the structural system

\[
T = \alpha H^\beta
\]  

(1)

It appeared in ATC3-06 (ATC, 1978) first with \( \beta \) equal to 0.75, while \( \alpha \) was calibrated as 0.06 (if H in measured in meters or 0.025 if it is in feet), based on periods measured on some buildings during the 1971 San Fernando earthquake. A similar relationship may be computed via the Rayleigh method (Chopra, 1995) with the following seismic design assumption: (i) horizontal forces linearly distributed along the height of the building; (ii) mass distribution constant along the height, (iii) linear deformed shape; (iv) base shear proportional to \( \gamma T/1 \). If these conditions are met the period is expressed as:

\[
T = \alpha H^{1/(2-\gamma)}
\]  

(2)

If \( \gamma \) equals 2/3, as established in US codes (UBC, 1997):

\[
T = \alpha H^{0.75}
\]  

(3)

In SEAOC-88 commentary (SEAOC, 1998) \( \alpha \) is 0.073 (if H in measured in meters or 0.030 if it is in feet). The formulation with these values of the parameters was adopted by International codes as Eurocode 8 (EC8, 2004) rounding \( \alpha \) to 0.075. Alternatively, NEHRP-94 (1994) includes a relationship as a function of the number of storeys (N), \( T = 0.1N \), limited to buildings up to 12 storeys with inter-storey height not smaller than 3m. This relationship was frequently adopted by codes before Eq. (1).

More recently, calibration of coefficients is based on experimental data; e.g. the monitoring of buildings during earthquakes. Goel and Chopra (1997), collected data on 37 reinforced concrete buildings, featuring seismic design and with height ranging from 10m to 100m. For each of the building the periods in the two principal directions were measured; in particular, the periods in the two directions are very similar, showing an average 10% difference, as shown in Figure 1. This may most likely be attributed to earthquake resistant design of the buildings, which should give uniform lateral stiffness in both directions.

Note that this kind of approach renders, of course, the period estimation depending on the history of the shaking at the site for each structure for two reasons: (1) if the shaking is strong enough to crack the structure the period measured is longer than if the structure remains uncracked; (2) if the buildings are subjected to multiple earthquakes the period measured after
the first cracking ground motion is always related to cracked stiffness also in subsequent lower intensity shaking.

The buildings of that study were subjected to 8 main Californian earthquakes from San Fernando (1971) to Northridge (1994). According to the authors, 22 buildings experienced a peak ground acceleration (PGA) lower than 0.15g, while the others were subjected to larger acceleration at the base. As expected, the latter buildings show a larger period given height.

Comparing experimental results to those deriving from Eq. (3) a underestimation of the period is observed, especially for larger height buildings and for those which experienced a PGA larger than 0.15g. Therefore alternative formulas were proposed resulting from a semi-empirical analysis. One, Eq. (4), features the best fit coefficient for Eq. (1); Eq. (5), is that fitting data plus one standard deviation; and Eq. (6) is that conservatively works at minus one standard deviation and, therefore, is proposed for estimation of the period in seismic design.

\[
T = 0.052H^{0.9} \quad (4)
\]

\[
T = 0.065H^{0.9} \quad (5)
\]

\[
T = 0.044H^{0.9} \quad (6)
\]

A similar study was carried out by Hong e Hwang (2000) for 21 RC seismic buildings in Taiwan subjected to 4 events claimed to not yield the structures. The coefficients proposed in that study lead to the following expression:

\[
T = 0.029H^{0.804} \quad (7)
\]

Comparing semi-empirical relationships significant differences in estimations are found. In Figure 2 such comparison is given, and it may be noted how the estimation according to Eq. (4) leads to an 130% average overestimation in respect to Eq. (7). Such a difference may be related with the different design criteria and construction practice in the two countries. Closer agreement is found between code-suggested relationships (Eq. (3), Eq. (6) and that function of

![Figure 1. Correlation of elastic periods measured in the two main directions of the buildings of the study from Goel and Chopra (1997).](image)

![Figure 2. Comparison of semi-empirical relationships for estimation of the period in seismic design.](image)
the number of storeys) for $H \leq 40\text{m}$, which is the range of interest for the building stock in southern Europe (e.g., Italy).

From the comparison it may be argued that the calibration for Eq. (1) via a numerical or experimental approaches is conditioned on assumptions on the dynamic response and on peculiarities of seismic design. It is therefore to investigate whether period-height relationships for sub-standard seismic design or gravity loads design buildings could lead to different estimation in respect that of codes and/or existing literature. These may be common conditions for existing buildings and, therefore, it is the focus of the following analyses.

![Figure 2. Literature period-height relationships](image)

3 SUB-STANDARD RC BUILDINGS

Existing RC buildings do not reflect regularity of strength and stiffness which a building conceived with capacity-design principles shows. In fact, most of them may be considered sub-standard being designed for gravity loads only in areas subsequently considered seismically hazardous or designed with inadequate seismic provisions. Herein samples from both categories are considered, and is to underline that may be significant structural differences even among them. Design for gravity loads does not require regularity of the structural system in the plan view and therefore these buildings show disuniform distribution of the resisting substructures which may lead to different elastic responses in the two main directions; in other word, the number and orientation of frames is determined by elements carrying gravity loads. Building designed also accounting for horizontal forces show a more distribution of frames because the action was assumed equal (and period-independent) in the two directions. This lead to a first three-dimensional conception of the structure with frames specifically devoted to resist seismic actions.

Within this framework load design was carried out with approximate methods. In the gravity-load design simplified schemes instead of plane frames were used. In fact, columns were proportioned based on the axial load only, while the beams’ design reflect the continuous multiple-supports scheme.

On the other hand, seismic load effects are evacuate via frame modeling, although the distribution of seismic actions to elements is still approximated being based on the floor masses’ distribution or on the columns’ inertia, the latter known as shear-type model. Seismic actions during the considered time span was determined imposing horizontal forces
depending on fixed acceleration levels: 0.10g or 0.05g (increased to 0.07g later on) depending on the assumed seismic potential of the site; in other words, period-independent seismic forces. Design criteria did not consider limit-states expressed in terms of maximum inter-storey drift ratios as modern codes, therefore, the storey-stiffness indirectly depends on the lateral load resistance which should be provided as a consequence of the horizontal forces above.

Therefore the following structural features are expected for this kind of buildings:

- existing buildings, independently of gravity loads or seismic design, feature a lower lateral stiffness in respect to modern code-designed structures;
- among existing buildings, those provided of seismic design have a larger global stiffness in respect to those designed for gravity-loads only as horizontal forces should imply larger dimensions of elements constituting the seismic resisting systems (i.e., frames);
- the eventual variability of structural system in gravity load design may lead to significant differences in the natural period in the two principal directions of these kind of buildings.

These issues are investigated in the following referring to a widely diffused typology of existing buildings in Italy built in the year approximately between 1950 and 1975.

4 METHODOLOGY

To investigate the elastic dynamic features of the buildings describe above as a function of various types of mechanical parameters was based on the simulated design of a population of buildings which is made of the following phases (Verderame et al., 2008):

- building definition. Depends only on the 3D dimension of the building;
- identification of possible structural system for the building. At this stage the structural parameters number of frames, bay lengths, column orientation are defined;
- simulated design of the structural systems in terms of cross sections’ dimensions and reinforcements both longitudinal and transversal of the elements;

In the case of gravity loads design the number of frames is determined by the distribution of vertical forces only, while in the case of seismic design additional frames derive from the lateral forces. The design of the elements is carried out referring to the recommendations of the codes enforced at the time in terms of reinforcement ratios, material design strengths and so on. Note that, steps (ii) and (iii) lead to multiple structures being associated to a single buildings as several structural systems and design alternatives correspond to the same global dimensions.

4.1 Considered buildings’ population

Considered buildings refer to a rectangular plan shape and a moderate number of storey. In this type of buildings typically there are one or two units for each floor and one a stairway which is assumed centered in respect to the longitudinal direction of the building (the longer). In generating the population of analyzed buildings the considered variable parameters are its longitudinal length ($L_x$), transversal length ($L_y$) and the global height ($H$) excluding the foundations. Interstorey height is constant and equal to 3.0m.

Structural configuration adopted in simulation refers to structural designed for gravity loads only subsequently adjusted (adding frames) to also withstand horizontal actions. Its principal feature is that frames, to support the slabs, have all the same direction, which is typically the transversal one of the building, this lead to call this resisting system as parallel plane frames.
In the transversal direction, then, only two frames at the two ends of the building and the stair module exist. In Figure 3a this scheme is illustrated. On the other hand the structural configuration in the case of seismic design corresponds with an integration of the former with multiple frames in the same direction, one per bay (Figure 3b).

For this kinds of buildings the bay lengths have been assumed to be comprised in the 3.0-5.0m; this lead to the variability of the structural configuration among the population.

Gravity loads design was carried out according to the design values of dead and live loads of the codes in the time-span considered. As discussed design of elements refers to simple substructuring schemes in which the design of columns is driven by axial load and design of beams refers to a multiple supports model.

Seismic design was carried out by static linear analysis, the most common tool at the time (and still widely used today in the practice). Three accelerations are considered for seismic design equal to 0.10g, 0.07g e 0.05g according to the evolution of seismic classification of the territory in Italy up to 1975. The seismic forces distribution considered is that proportional to the storey masses (R.D.L. 640, 1935; R.D.L. 2105, 1937; Legge 1684, 1962). The statical location of horizontal forces refers to the flexible-slab assumption; in the transversal direction the contribution of the stair is not considered.

The ranges of variability of building dimensions considered is that as follows:
- longitudinal length \( L_x = [15.0, 20.0, 25.0, 30.0] \text{m}; \)
- transversal length \( L_y = [8.0, 10.0, 12.0] \text{m}; \)
- building height (H) is comprised between (6.0÷24.0)\text{m} corresponding to 2-8 storey

All possible combinations of these values lead to 84 buildings for each of the two possible design categories (seismic and gravity loads); while the structural configuration variability lead to 175 structural systems, two per building on average.

The allowable stress design leads to proportions of the elements which may be defined as minimal in respect to the actions induced by gravity and seismic loads. Therefore the lateral stiffness is generally to be considered minimal in the two directions.

Considering also the four design options (one gravity load and three seismic design levels) a population of 700 structures was analyzed. The cylindrical compressive mean strength of concrete \( f_c \) is constant among the structures and equal to 15 MPa.

In particular two linear analyses have been carried out for each structure to investigate the elastic period in the two principal directions of the buildings to which the structure considered correspond to.

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![Figure 3. Structural configurations of the considered building types: gravity-loads designed (a) and seismic design (b).](image-url)
5 IMPLICATIONS OF STRUCTURAL FEATURES ELASTIC PROPERTIES

The elastic characteristic of the different directions of each considered structures are evaluated via the classical eigenvalues analysis, i.e., Eq. 8:

\[
[K] - \omega^2[M] \{\phi\} = \{0\}
\]

where \([K]\) is the stiffness matrix of the MDOF structural system, \([M]\) is the seismic storey masses matrix, \(\{\phi\}\) is the displacement vector of vibration mode, and \(\omega\) is the associated circular frequency. \([K]\) is determined starting from the cross section stiffness (EcI), which has been evaluated referring to the inertia (I) of the uncracked section and the elastic modulus of concrete, Ec, defined as in Eq.9 in which \(f_c\) is the cylindrical compressive strength of concrete expressed in MPa (EC2, 2004).

\[
E_c = 22000(f_c / 10)^{0.3} \text{ (MPa)}
\]  

The storey masses in the diagonal \([M]\) matrix have been evaluated according to Eurocode 8 (EC8, 2004) based on the analysis of dead and live loads for the structure for which the elastic periods are evaluated.

From Eq.8 the \(\omega_i\), \(\{\phi\}_i\) and \(m^*_i\), associated to the i-th mode defined as the effective mass and evaluated as:

\[
m^*_i = \sum m_k \phi_{k,i}
\]

where \(m_k\) is the seismic mass of the k-th storey, \(\phi_{k,i}\) is the displacement of the k-th floor in the i-th mode.

The periods considered are those corresponding only to the fundamental modes, primarily translational, associated to the principal directions of the structure. Therefore, for each of the two directions the fundamental \(\{\phi\}\), \(\omega\), and \(m^*\) are determined and the corresponding elastic period, \(T_{el}\), defined as:

\[
T_{el} = \frac{2\pi}{\omega}
\]

Figure 4 shows the trends of the periods as a function of the height for the four categories of buildings investigated. As expected the design options affect the elastic periods. The periods corresponding to gravity load design are generally smaller than those proportioned to resist also to horizontal force, among which to a higher reference acceleration correspond lower natural periods. Moreover the longitudinal direction is generally stiffer than the transverse, although the transverse to longitudinal periods ratio is larger in the case of design for gravity loads in respect to seismic.

5.1 Gravity loads designed buildings

In these structures the period in the short direction is more variable in respect to that longitudinal given height. This depends on the peculiar structural configuration, Figure 3a, which significantly affects, the ratio between effective mass \(m^*\) and lateral stiffness, \(K_{el}\), of the building. In fact, in Figure 5, the trends of these quantities are given as a function of the height for the two directions.
Elastic Period of Existing RC-MRF Buildings

The effective mass varies almost linearly with the height, and it is variable in both directions because of the ranges of global dimensions, $L_x$ and $L_y$, considered which lead to a wide range of plan areas for the populations analyzed, as it can be observed in Figure 6a. In general, the buildings feature effective masses similar in the two directions, therefore the differences in elastic periods depend on the different lateral stiffness.

As observed in Figure 5, the long direction has lateral stiffness larger than that in the short direction, this discrepancy is increasing with height and is up to 50% for buildings with more than 3 storeys. Vice versa, the longitudinal direction has a larger variability of the stiffness in respect to the transversal one. This has to be attributed to the different lateral resisting system of the two directions (Figure 3a).

The stair sub-structure significantly affects the stiffness in the short direction. Despite this direction has only the two perimetral frames, this is magnified reflected for buildings with 3 storeys or less (H equal or less than 9m), for which the transverse stiffness is larger than the longitudinal. The stair module effect, rapidly decreases with height.

The variability of the area covered by the buildings is differently reflected in the elastic properties of the two directions (Figure 6b). In the longitudinal direction the increase of $L_x$ implies an increase in the number of bays, while an increase in $L_y$ lead to increase the number of longitudinal frames. Therefore, the longitudinal direction has an increasing stiffness with
area. Vice versa, the resisting system in the transversal direction is only marginally affected by an increase in the area. In fact, it does not imply modification in the stair module and the slight trend observed is due to the variation of number of bays or of the proportions of the elements of the two perimetral frames.

It may be concluded, that for gravity load buildings, the stair module plays a determinant role in the transversal period determination. The lower stiffness in the short direction in respect to the longer and its comparatively small variability given height clarify the trend of the period reported in Figure 4a.

![Graphs showing transverse and longitudinal elastic stiffness and effective mass versus height.](a) Transverse elastic stiffness versus height; (b) Longitudinal elastic stiffness versus height; (c) Transverse effective mass versus height; (d) Longitudinal effective mass versus height.

5.2 Seismic buildings

Seismic building have a defined resisting system also in the transversal direction and are also characterized by a more uniform distribution of structural sub-systems, e.g., Figure 3b. This reflect in different trends of stiffness in respect to gravity loads designed buildings.

The stiffness in the two direction is similar. In fact the stiffness in the longitudinal direction is never larger more than 20% with respect to that transversal. In particular, 2 storey buildings have a stiffness constant, on average, in respect to the design acceleration. The moderate height leads to actions that can be taken by the minima proportions of structural elements.
Elastic Period of Existing RC-MRF Buildings

Therefore, the structures result generally similar independent of the design acceleration. Moreover, for this kind of buildings the stiffness reduces with global height in both directions and variability is also comparable. The more uniform distribution of resisting systems in respect to gravity-load designed also reflects in a different trend of stiffness as a function of plan area which now affects also the transversal direction. In Figure 7a and 7b $K_{el}$ is given for the two directions of buildings with $H$ equal to 15.0m and designed for 0.05g and 0.10g respectively.

Seismic designed buildings are generally stiffer that those designed for gravity loads only. In Figure 8 the ratio of the average seismic to gravity loads stiffness is given for the two directions. This ratio is increasing in the transversal direction while it is almost constant in the longitudinal. This is mainly because the different structural systems is shown main in the former direction rather than in the latter. The plane frames added in the seismic design contributes to the stiffness in an increasing manner with respect to the height of the building. In the longitudinal direction, an increase in the design accelerations also implies a increment in the stiffness leading, on average, to a ratio in respect to the gravity-loads design case of
1.25, 1.35, and 1.45 for 0.05g, 0.07g and 0.10g. In the transverse direction, 20% is the minimum stiffness increment, a number found for 2 storey buildings, while 95% is the scored by 8 storey buildings, corresponding to the maximum global height considered in the study, and designed for 0.10g.

These results reflects in the trends of Figure 4b, 4c, 4d.

![Figure 8. Average seismic to gravity elastic stiffness ratio in transverse (a) and longitudinal (b) direction](image-url)

### 6 PERIOD PREDICTORS FROM REGRESSION ANALYSIS

Although, not an experimental sample, on the analyzed populations simple regression analysis allowed to see how results in terms of fundamental period display in respect to height. For comparative purposes, the same power-law formulation of Eq. (1) was assumed and the coefficients estimated via ordinary least square regression.

For the case of gravity load design the relationships for transverse and longitudinal directions are respectively:

\[
T_{el} = 0.076H^{0.93} \quad T_{el} = 0.135H^{0.67}
\]  

(12)

Moreover, for the transverse direction, the same relationship was retrieved also not considering the contribution of stair sub-structure, Eq. (13).

\[
T_{el} = 0.105H^{0.94}
\]  

(13)

Analogous relationships were determined for the three populations of seismic buildings:

*design acceleration 0.05g*

\[
T_{el} = 0.091H^{0.79} \quad T_{el} = 0.112H^{0.69}
\]  

(14)

*design acceleration 0.07g*

\[
T_{el} = 0.098H^{0.75} \quad T_{el} = 0.118H^{0.66}
\]  

(15)

*design acceleration 0.10g*
\[ T_{\text{el}} = 0.107H^{0.70} \quad T_{\text{el}} = 0.118H^{0.65} \]  

(16)

Figure 9 serves to compare the relationships found for the two considered directions. The trends generally reflect what observed for stiffness. The largest periods is observed for gravity-loads designed buildings while it decreases if the design acceleration is increased. In the longitudinal direction the period of seismic buildings is lower by 10 to 20% in respect to gravity loads. In the transversal direction, the reduction in period or seismic buildings may be as large as 45% for 8 storey buildings. Comparing the period-height relationship including and not-including the stair module for gravity load design buildings, the contribution of the sub-structure may be appreciated as the stir les to a reduction of the latter as large as 40%.

Finally, only as a reference the EC8 (EC8, 2004) period-height curve is also given in the figure. The EC8 period is systematically lower than what found because it is a lower bound itself (Goel and Chopra, 1997) and also because it is expected to refer to buildings featuring a different design philosophy.

![Figure 9. Comparison of period-height relationships for the buildings analyzed in the transverse (a) and longitudinal (b) directions.](image)

Because Figure 6 and 7, show that the effective mass and translational stiffness is also correlated with the plan extension of the building is expected that this variable has some prediction power in respect to the period. Therefore, an expression which includes also the plan area is considered Eq. (17):

\[ T = \alpha H^6 S^7 \]  

(17)

where \( S \) is the product of the two principal plan dimensions of the building \( L_x \) and \( L_y \). Least squares regression lead to the relationships for the transversal and longitudinal directions respectively given in Eq. (18):

\[ T_{\text{el}} = 0.009H^{0.33}S^{0.39} \quad T_{\text{el}} = 0.044H^{0.67}S^{0.21} \]  

(18)

the same kind of analysis for the seismic directions provides:

*design acceleration 0.05g*
\[ T_{el} = 0.029H^{0.79}S^{0.21} \quad T_{el} = 0.059H^{0.69}S^{0.14} \]  

*design acceleration 0.07g*

\[ T_{el} = 0.033H^{0.75}S^{0.20} \quad T_{el} = 0.062H^{0.66}S^{0.12} \]  

*design acceleration 0.10g*

\[ T_{el} = 0.039H^{0.70}S^{0.19} \quad T_{el} = 0.068H^{0.65}S^{0.10} \]

How much S contributes to explain the period is assessed simply by analyzing the standard error for the regressions’ residuals, \( \sigma_T \):

\[
\sigma_T = \sqrt{\frac{\sum (\log T - \log T_i)^2}{n-1}}
\]  

In Eq. (22) \( T \) is the period from the regression model for the building having the computed value \( T_i \) and \( n \) is the size of the sample. In Table 1 the values of \( \sigma_T \) for all cases analyzed are reported, for the two conditions of including S along to H or not. Results lead to conclude that only for gravity load design S add information on the period as for this building typology adding S reduces the standard lead to a 60% reduction of the standard error in respect to formulation which only account for the period height.

### Table 1. Standard error for the regressions’ residuals.

<table>
<thead>
<tr>
<th>Design type</th>
<th>direction</th>
<th>standard error, ( \sigma_T )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( T=\alpha H^\beta )</td>
</tr>
<tr>
<td>Gravity</td>
<td>transverse</td>
<td>0.131</td>
</tr>
<tr>
<td></td>
<td>longitudinal</td>
<td>0.078</td>
</tr>
<tr>
<td>Seismic 0.05g</td>
<td>transverse</td>
<td>0.086</td>
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<tr>
<td></td>
<td>longitudinal</td>
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<tr>
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<tr>
<td></td>
<td>longitudinal</td>
<td>0.066</td>
</tr>
<tr>
<td>Seismic 0.10g</td>
<td>transverse</td>
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<tr>
<td></td>
<td>longitudinal</td>
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</table>

### 7 CONCLUSIONS

The elastic period has a primary role in the seismic assessment of buildings. Main codes propose simplified equations, retrieved on semi-empirical basis, expressing the fundamental period as a function of height, which represents the relationship between mass and stiffness of the structure. Nevertheless, the most of these relationships are based on data of buildings reflecting seismic design criteria very different from those of the European existing structures. In the study presented two populations of reinforced concrete buildings have been investigated: the first one being designed for gravity load only, the second one designed with obsolete seismic design criteria. Modal analyses allowed to assess the influence of design
criteria, structural system and global dimensions (area and height) on the elastic stiffness, the effective mass, and the elastic period for both principal axes of the building.

Results are different for the two classes:

- Gravity load design buildings feature periods in the two directions which have an increasing difference with height and as large as 50%. The period shows large variability in the short direction due to the variability of plan area.
- Seismic design buildings show a lower period in the longitudinal direction with respect to the corresponding gravity load buildings; this reduction, obviously, increases with design acceleration and is up to 20%. In the short direction the reduction of fundamental period is more significant (50%) because it is due not only to different design criteria, but also (mainly) to the different structural system.

Finally, based on the results of the analyses a power-law regression was carried out as a function of height. In the comparison with EC8 formulas existing buildings show systematically larger periods. In particular, gravity load designed buildings, featuring a 3D structural system, seem to require a twofold definition of period referring to the two directions. Therefore, height alone is inadequate to explain period variability. In fact, also a global parameter (e.g., plan area) should be added in simplified relationships for rapid period evaluation.

8 ACKNOWLEDGEMENTS


9 REFERENCES

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