AN UPDATED MODEL OF EQUIVALENT DIAGONAL STRUT FOR INFILL PANELS

Giuseppina Amato, Marinella Fossetti, Liborio Cavaleri, Maurizio Papia

Università di Palermo, Dipartimento di Ingegneria Strutturale, Aeroespaziale e Geotecnica, Italy,
cavaleri@diseg.unipa.it

ABSTRACT
Recently the interest of the scientific community in infills in framed structures has increased, recognising their non-negligible effects on the primary structure. Although different codes, as a reflex, stress the need to take the infills into account in the analysis of framed structures, often this invitation does not correspond to proper and detailed code rules. In this connection, in this paper, the European and American codes are discussed, stressing that a defined and appropriate approach to this problem is missing in them. Further, a simplified tool for the evaluation of the effects of the infills is suggested, to be adopted by technical codes and used in practical applications. This approach, recalling the well-known equivalent strut model, is based on the usual assumptions, but considers some aspects of the problem of the frame-infill interactions, which are recognised as basic in the literature but not included in the available models.

KEYWORDS
Infills, framed structures, equivalent pin-jointed strut.

1 INTRODUCTION
Today it is generally accepted that infills modify the behaviour of framed structures under lateral loads; nevertheless, infills are generally neglected in structural analysis, and this can lead to an unreliable evaluation of the response in several cases. Considering only the principal effects, it can be observed that: - the infills modify the lateral stiffness of the infilled frames, and, consequently, the modal characteristics of a framed building, producing a change in the base excitation-structure interactions; - a non-uniform infill distribution in plan and in elevation can produce torsional effects and soft storeys, respectively, with possible premature collapse in the case of a severe earthquake; - infills can produce increased shear stresses on the columns that may modify the mode of collapse due to reduction in structural ductility.
Although the negative effects of infills are generally stressed, it is also true that uniformly distributed infills give a lateral strength contribution, which can prove to be useful in the case of weak frames, not designed following seismic capacity criteria.
To sum up, infills have to be considered in structural analysis, because they can change the safety level of bare structures, making apparently safe structures unsafe and apparently unsafe structures safe.
A successful way to take infills into account in framed structures is based on their substitution with a couple of equivalent pin-jointed struts, alternatively effective in relation to the
direction of the lateral loads. This approach is also suggested by some technical codes, but the problem of the definition of the strut characteristics is not adequately treated (the definition of the geometrical and mechanical characteristics of this equivalent element is a question that each designer must solve before a reliable structural analysis is carried out) or non-updated models are proposed, which do not take into account recent advances in research on this topic. If, on one side, technical codes give insufficient or inappropriate indications, on the other side it is not possible to find a univocal approach to the problem of infills in the literature. By contrast, one can find several approaches, based on different analytical, numerical or experimental investigations, leading to very different results.

For example, by using the historically first approach proposed by Stafford Smith, 1966, for evaluating the width of the equivalent strut, one finds a value that is very different from those deducible from subsequent researches (e.g. Stafford Smith and Carter, 1969, Mainstone, 1971, 1974, Klingner and Bertero, 1978, Durrani and Luo, 1994). Many more details on this aspect can be found in Papia et al., 2003.

This discrepancy is due to the fact that the stiffening effect of an infill depends on many parameters (mechanical characteristics of the infill, stiffness ratio between frame and infill, etc) that have usually been considered, but it is also influenced by some other parameters, whose effect has been recognised by the scientific community but is not included in the proposed models: axial stiffness of columns of the frame, vertical load occurring after the construction of the infills, transverse strain ratio of the material constituting the infill panel.

Stafford Smith, 1966, first stressed the effect of vertical loads, on the basis of experimental observations; nevertheless, his calculus tool does not allow a generalized model including this effect. Subsequently, a specific resolution adopted during the NCEER workshop on Seismic Response of Masonry Infills (1994) stated that vertical loads have to be taken into account for the identification of the equivalent strut. In Papia et al., 2004 and Amato et al., 2008a the effect of vertical loads is once again discussed together with those produced by the further parameters mentioned above, previously considered in Papia et al., 2003.

In this framework, this paper proposes an updated general model of pin-jointed equivalent diagonal strut, which could be utilized for practical applications and suggested by technical codes, by virtue of its simplicity of use.

2 PROVISIONS GIVEN BY EC8 AND FEMA 356

As mentioned above, many technical codes recognise the need to take the infills into account in the evaluation of the response of infilled frames. Eurocode 8 and FEMA 356 can be mentioned among the most representative codes for the extension of the territories in which they are in force and because they are considered a reference for many others codes. For this reason their content regarding the effect of infills will be commented on here.

2.1 Eurocode 8

In the section devoted to modelling in structural analysis of the last version of Eurocode 8 (UNI EN 1998-1:2004), one reads: *infill walls which contribute significantly to the lateral stiffness and resistance of building should be taken into account.*

Then, in the section regarding irregularities in plan, it is stated that: *infills should be included in the model and a sensitivity analysis regarding the position and the properties of the infills should be performed.* Then, with reference to non uniform distribution of infills in elevation, *if a more precise model is not used,* one can calculate the seismic action effects on columns by amplifying them by a magnification factor.
Hence, many times the use of a reliable model is recommended. Nevertheless, no models for the infill are included in Eurocode 8 as a support for practical applications, leaving designers free in choosing a criterion for modelling infills and identifying the complex frame-infill interactions.

2.2 FEMA 356

Unlike Eurocode 8, the Federal Emergency Management Agency (FEMA) code 356 explains clearly enough how to take infills into account: the effect of infills has to be considered by a FEM analysis or, alternatively, by introducing a diagonal pin-jointed strut equivalent to the infill.

For the first option no more is said, unlike the second one, which is derived from an experimental observation: under lateral forces the frame tends to separate from the infill near the windward lower and leeward upper corners of the infilled mesh.

For FEMA 356 the equivalent strut is to have the same thickness and modulus of elasticity as the infill panel (but it is not clear along which direction the modulus of elasticity must be calculated) while the width $w$ is given by the following equation:

$$\frac{w}{d} = 0.175 (\lambda' h')^{-0.4}$$  \hspace{1cm} (1)

where

$$\lambda' = \sqrt{\frac{E_d t \sin(2\theta)}{4 E_f I_c h}}$$  \hspace{1cm} (2)

in which $t$ is the thickness of the infill, $h$ and $\ell$ are its height and length, respectively, $\theta = \text{atan}(h/\ell)$, $I_c$ is the moment of inertia of the columns, $h'$ is the height of the frame, measured between the centrelines of the beams; $E_d$ and $E_f$ are Young’s modulus of the infill and of the material constituting the frame, respectively.

![Figure 1. Geometric characteristics in Eqs. (1) and (2).](image)

Eq. (1) was proposed in Mainstone, 1974, to identify the mean lateral stiffness of the infilled frame before the cracking of the infill.

The equivalent strut can be modelled as a concentric element connecting the intersections between beams and columns. This scheme does not give evidence of the local effects produced by the infill on beam and column regions near the nodes. For this reason, although
the force acting on the strut is evaluated with concentrically located struts, it must be considered to be acting eccentrically, in agreement with the schemes in Figure 2.

\[ w \times \cos \beta \]
\[ w \times \sin \beta \]

**Figure 2. Schemes with eccentric reaction of strut.**

Finally, the strength of the strut can be simply obtained as a projection of the shear strength of the infill, as specified at section 7.5.2.2 of FEMA 356. Although FEMA gives many more details about the model to be used for the infills, some aspects of the frame-infill interaction are not treated: for example, the effect of vertical loads, which modify the capacity of the infills to stiffen the surrounding frames because of a different length of contact between frame and infill under a lateral load (Figure 3). This effect should lead to a greater value of \( w \).

**Figure 3. Influence of vertical load on effectiveness of infill.**

Another aspect of the problem, which has been observed by means of FEM analyses and experimental tests, is the influence of the transverse strain ratio (Poisson’s ratio) of the material constituting the infill. High values of this parameter produce an extension of the frame-infill contact region because of the transverse dilatation of the diagonally compressed infill panel. It follows that the width of the equivalent diagonal strut should be higher.

As a further remark, the modulus of elasticity of the infill inserted in Eq. (2) should be referred to the diagonal direction. The evaluation of this modulus in practical applications is a problem that should be specifically addressed.

What has been said highlights that the model assumed by FEMA is not wholly appropriate, since the calculus tools that were available when Mainstone formulated the model were not so
powerful as the present ones. Therefore, the use of modern calculus tools should be encouraged for an improvement of the models proposed by the codes in force. In the next sections, an attempt is made to show the inappropriateness of Mainstone’s model and an implemented model is proposed for practical applications.

3 PROPOSAL FOR IDENTIFYING EQUIVALENT PIN-JOINED STRUT

A tool for the identification of the strut equivalent to the infill belonging to the generic mesh of a framed structure can be simply obtained by extending the study of a single-bay single-storey infilled frame. Referring to this structural system, the section of the equivalent pin-jointed strut can be identified by imposing the condition that the initial stiffness of the actual system is equal to the initial stiffness of the equivalent braced frame (Figure 4).

\[
D_i = \frac{k_d \cos^2 \theta}{1 + \frac{k_d}{k_c} \sin^2 \theta + \frac{1}{4} \frac{k_d}{k_b} \cos^2 \theta} + 24 \frac{E_f I_b}{h'^3} \left( 1 - 1.5 \left( \frac{3}{I_c} \frac{h'}{\ell'} + 2 \right) \right)^{-1}
\]

(3)

where \( k_d, k_c, k_b \) are the axial stiffnesses of the equivalent strut, of the column and of the beam, respectively:

\[
k_d = \frac{E_d t w}{d}; \quad k_c = \frac{E_f A_c}{h'}; \quad k_b = \frac{E_f A_b}{\ell'}
\]

(4)

In Eq. (4), \( I_b \) and \( A_c \) are the moment of inertia and the area of the cross-section of the columns, while \( A_b \) is the area of the cross-section of the beam.

The stiffness \( D_i \) of the actual system can be evaluated by a FEM analysis taking into account the level of the vertical load and the frame-infill interaction along the contact surface. In the present work the contact surface was modelled by using finite elements with no tensile strength and able to transmit frictional shear stresses proportional to the compression stresses. In this way it was possible to include the effects of the frame-infill detachment produced by the lateral load.

The value of the width \( w \) can be calculated by imposing the condition that

\[
D_i(w) = \tilde{D}_i
\]

(5)
Repeating this procedure for different characteristics of infill and surrounding frame a set of widths can be obtained.

It was recognised that these data can be related to a parameter, denoted as \( \lambda^* \), which depends on some mechanical characteristics of frame and infill and has a similar meaning to the parameter \( \lambda' \) introduced by Mainstone (the details about the identification of \( \lambda^* \) as a parameter can be found in Papia et al, 2003). Further relations can be obtained as a function of the level of vertical load, of Poisson’s ratio and of the aspect ratio of the infill.

Considering that the parameter \( \lambda^* \) is expressed by

\[
\lambda^* = \frac{E_d \ t \ h'}{E_f \ A_c \left( \frac{1}{\ell'^2} + \frac{A_c}{4 A_b h'} \right)}
\]

and that the level of the vertical load \( F_v \) is expressed by the dimensionless parameter

\[
\varepsilon_v = \frac{F_v}{2 A_c E_f}
\]

the numerical experimentation gave the results partially contained in Figure 5.

![Figure 5. Dimensionless width of equivalent strut cross-section for different infilled frames.](image)

The results highlight the effect of the vertical loads and of Poisson’s ratio on the behaviour of an infilled frame and, consequently, on the identification of the width of the equivalent strut cross-section.

The points that were obtained numerically can be fitted by the following equation:

\[
\frac{w}{d} = k \frac{c}{z (\lambda^*)^\beta}
\]
where \( c \) and \( \beta \) depend on Poisson’s ratio \( \nu \) of the infill masonry along the diagonal direction,

\[
  c = 0.249 - 0.0116 \nu + 0.567 \nu^2
\]  

(9)

\[
  \beta = 0.146 + 0.0073 \nu + 0.126 \nu^2
\]  

(10)

the parameter \( z \) is a function of the aspect ratio of the infill,

\[
  z = 1 + 0.25 (\ell/h - 1), \quad 1 \leq \ell/h \leq 1.5
\]  

(11)

while \( k \) is a function of \( \varepsilon_v \) and \( \lambda^* \),

\[
  k = 1 + (18\lambda^* + 200)\varepsilon_v
\]  

(12)

Figures 6-7 show the curves obtained by Eq.(8) for two values of Poisson’s ratio and of the aspect ratio of the infill.

A comparison was made between the results of the numerical investigation described here and the results obtainable by means of Mainstone’s model (Eq. (1)).
For this comparison, because of the different expression of $\lambda'$ and $\lambda^*$, an ad hoc procedure was followed: once an infilled frame was fixed, the values of $\lambda'$ and $\lambda^*$ were calculated; then, the corresponding value of $w/d$ was obtained by using Mainstone’s model, and this value of $w/d$ was plotted versus the parameter $\lambda^*$. The above sequence was repeated for different infilled frames.

Figure 8 shows the results of this procedure. It must be observed that the effect of vertical loads, not included in the model proposed by Mainstone, 1974, has not been considered.

![Figure 8. Comparison between proposed model and model adopted by FEMA 356 (t/h =1).](image)

By observing Figure 8, one notes that the values of $w/d$ obtainable using Mainstone’s model are much lower than those obtainable with the model proposed here. A similar underestimation of the width of the equivalent strut can be found after a comparison with some other models available in the literature, as discussed in Papia et al., 2003.

The distance between Mainstone’s model and the model proposed here increases when a level of vertical loads different from zero is considered.

4 VALIDATION OF PROPOSED MODEL

Some experimental tests made it possible to validate the model that is proposed here. Single-bay single-storey infilled frames were tested for the measure of their lateral stiffness. The specimens were subjected to a constant value of vertical load (400 kN) and to a monotonically increasing lateral force. Figure 9 shows two specimens during the test.

In Table 1 some geometric and mechanical characteristics of the specimens that were tested are included, while the results in Table 2 allow comparison of the numerical and experimental values.

The experimental values of the lateral stiffness were calculated considering the first phase of the response, in which elastic behaviour was observed. The values of the elastic modulus and Poisson’s ratio of the infill along the diagonal direction were obtained starting from the results of experimental tests on masonry samples and by applying the procedure proposed in Amato et al., 2008b.

Comparison shows good agreement between the experimental stiffness $D_{ex}$, and the values of stiffness $D_i$ obtained by the proposed model, confirming the reliability of the model itself.
Figure 9. Tests on frames with different kinds of masonry infills.

<table>
<thead>
<tr>
<th>Frame</th>
<th>Kind of Infill masonry</th>
<th>$E_f$ (kN/mm$^2$)</th>
<th>Cross-section of columns (mm$x$mm)</th>
<th>Cross-section of beam (mm$x$mm)</th>
<th>Height and width of infill (mm, mm)</th>
<th>Thickness of infill (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>calcarenite</td>
<td>200x200</td>
<td>200x400</td>
<td>1600, 1600</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>clay brick</td>
<td>200x200</td>
<td>200x400</td>
<td>1600, 1600</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>lightweight concrete</td>
<td>300x300</td>
<td>300x400</td>
<td>1600, 1600</td>
<td>300</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Lateral stiffness of infilled frames: numerical and experimental values.

<table>
<thead>
<tr>
<th>Kind of Infill masonry</th>
<th>$E_d$ (N/mm$^2$)</th>
<th>$\nu$</th>
<th>$\lambda^*$</th>
<th>w/d</th>
<th>w (mm)</th>
<th>$D_i$ (kN/mm)</th>
<th>Specimen</th>
<th>$D_{ls}$ (kN/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>calcarenite</td>
<td>4292</td>
<td>0.30</td>
<td>1.55</td>
<td>0.27</td>
<td>610</td>
<td>110</td>
<td>1</td>
<td>125</td>
</tr>
<tr>
<td>clay tile</td>
<td>5303</td>
<td>0.07</td>
<td>1.43</td>
<td>0.25</td>
<td>565</td>
<td>122</td>
<td>3</td>
<td>128</td>
</tr>
<tr>
<td>lightweight concrete</td>
<td>1795</td>
<td>0</td>
<td>0.40</td>
<td>0.29</td>
<td>655</td>
<td>127.5</td>
<td>5</td>
<td>115</td>
</tr>
</tbody>
</table>

5 CONCLUSIONS

This paper shows that the influence of infills on the behaviour of infilled frames is not adequately treated by technical codes although infills may be basic for the structural response. In particular, while EC8 leaves the designer absolutely free, not giving specific instructions and models to be applied to include the presence of infills in the analysis model, FEMA 356 suggests the substitution of the infills with equivalent struts identified using the model proposed by Mainstone, 1974. This model fails to take some parameters into account, which
could substantially change the lateral response of an infilled frame; further, it seems to underestimate the stiffening effect of the infill.

In this context, confirming the criteria proposed by FEMA for the evaluation of the forces transmitted by the infill to the surrounding frame, a new model has been presented and proposed for identifying the equivalent strut, taking these parameters into account. It is the result of a numerical experimentation carried out with a FEM approach, in which the frame-infill interface has been adequately characterized mechanically.

The results obtainable with the proposed model were compared with the results of experimental test on infilled frames with different kinds of infills, confirming its reliability.

The proposed model makes it easy to identify the width of the equivalent strut thanks to an analytical expression containing the parameters mentioned several times.

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7 REFERENCES