CAPACITY MODELS OF RC MEMBERS WITH EMPHASIS ON SUB-STANDARD COLUMNS WITH PLAIN BARS

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ABSTRACT
Current code prescriptions allow to evaluate the ultimate rotational capacity from hybrid (mechanical-empirical) or empirical formulations, for R.C. members with deformed bars and seismically detailed. These formulations can be extended to non-conforming elements by applying correction coefficients calibrated on experimental data. These coefficients, for members with plain bars, imply a reduction of 40% at least; this reduction increases if lapping of longitudinal bars is present. The experimental campaign conducted at the University of Naples on 16 real-scale columns with plain bars allowed to extend the experimental database for this typology. Experimental results highlight the excessive conservativeness of the current code proposal. Based on these results, new correction coefficients are proposed.

KEYWORDS
Existing RC building, non conforming, ultimate capacity, plain bars, lapping.

1 INTRODUCTION
The present Italian technical regulations (D.M. 2008), on a level with the most modern of international codes (CEN, 2005), allow us to determine the seismic capacity of existing RC buildings with recourse to non-linear analysis methodologies. The use of such methods of analysis, however, requires knowledge of the real post-elastic rotational capacities of each element of the construction (beams, columns) both in monotonic field, for non-linear static analysis, and in cyclical field, for non-linear dynamic analysis. In monotonic field, a series of parameters (yielding, peak resistance, ultimate state) has to be defined, in order to define the response curve of the element. In cyclic field, hysteretic rules and strength and stiffness degradation models have to be defined; they significantly influence the assessment of ultimate rotational capacity. Nevertheless, these rules are not easy to define, due to the number of geometrical and mechanical parameters and to the uncertainties involved. For example, the type of loading influences in a not negligible way the response of the r.c. element. Most of the code prescriptions only define the deformation capacity at the elastic limit (yielding) and at ultimate (collapse); therefore, based on these prescriptions, it is not possible to completely
define the strength degradation of the monotonic envelope, nor the hysteretic behaviour through appropriate rules.

Generally, deformation at yielding is evaluated as a chord rotation, accounting for different contributions corresponding to bending, shear and fixed-end rotation deformation mechanisms.

The rotational capacity is generally evaluated referring to a fixed strength decay (20%) respect to the peak resistance, evaluated on the envelope force-displacement curve. It is clear that this definition is strongly influenced by the maximum resistance condition, as well as the post-peak degradation, monotonic or cyclic. It is difficult to define a relationship between the element parameters and the rotational capacity, due to the complex phenomena influencing the post-elastic deformation behaviour and to the natural variability affecting this phenomena.

The code, consistently with the methodologies developed in literature, proposes two main approaches: a mechanical-empirical approach, based on plastic hinge length concept, and a purely empirical approach.

Referring to the purely empirical formulation proposed in (CEN, 2005), in the present work, based on experimental data, the applicability of this formulation to under-designed elements with plain bars is evaluated. In particular, correction coefficients applied to the code formulation are proposed for elements with plain bars, with or without lapping of longitudinal reinforcement.

2 EVALUATION OF ULTIMATE CHORD ROTATION

In this section, the theoretical background of current European code (CEN, 2005) formulas for the ultimate rotational capacity of reinforced concrete members is presented. Principles and methodologies standing behind the two main approaches to the assessment of this value (mechanical and empirical) are introduced.

2.1 Code formulas (EC8 part 3.3)

Eurocode 8 – Part 3 at section A.3.2.2 (Limit state of near collapse) provides expressions for the evaluation of ultimate element capacity of R.C. elements. The value of total chord rotation capacity under cyclic loading, following a mechanical approach, is given by [EC8 - Eq. (A.1)]:

\[
\theta_{um} = \frac{1}{\gamma_{el}} \cdot 0.016 \cdot (0.30^\nu) \left[ \frac{\max(0.01;\omega')}{\max(0.01;\omega)} \right]^{0.225} \left( \frac{L_V}{h} \right)^{0.35} \left( \frac{\alpha_{pl}}{\gamma_{sy} f_c} \right) (1.25^{100/pd})
\]

where \(\gamma_{el}\) equal to 1.5 for primary seismic elements and to 1.0 for secondary seismic elements, is meant to convert mean values of chord rotation to mean-minus-one-standard-deviation ones. The code also provides another expression for the evaluation of the plastic part of the ultimate chord rotation [EC8 - Eq. (A.3)]:

\[
\theta_{um}^{pl} = \theta_{um} - \theta_y = \frac{1}{\gamma_{el}} \cdot 0.0145 \cdot (0.25^\nu) \left[ \frac{\max(0.01;\omega')}{\max(0.01;\omega)} \right]^{0.3} f_c^{0.2} \left( \frac{L_V}{h} \right)^{0.35} \left( \frac{\alpha_{pl}}{\gamma_{sy} f_c} \right) (1.275^{100/pd})
\]
In this expression the coefficient $\gamma_{el}$ equal to 1.8 for primary elements and to 1.0 for secondary ones. To evaluate the total chord rotation, the plastic part calculated according to this formula should be added to the yielding rotation [EC8 - Eq. (A.10)].

The values of chord rotation calculated according to (1) and (2) apply to elements with deformed bars, seismically detailed and without lapping of longitudinal bars in the vicinity of the end region where yielding is expected (plastic hinge region).

The correction coefficient applied to members with deformed bars without seismic detailing is equal to 0.825 for both formulas. If the longitudinal deformed bars are lapped, expressions (1) and (2) should be applied doubling the mechanical compression reinforcement ratio ($\omega'$).

Moreover, if the lap length is less than the minimum value $L_{\text{min},\text{ou}}$, another reduction factor equal to $(l_o/L_{\text{min},\text{ou}})$ should be applied, calibrated only for expression (2), that is only for the plastic part of chord rotation. Corrections applied to the yielding chord rotation are given at section A.3.2.4(3) of the code; they are omitted here for the sake of brevity.

In elements with plain bars the chord rotation evaluated according to (1) should be multiplied by 0.575, while the plastic part of chord rotation given by (2) should be multiplied by 0.375. It’s worth noting that both coefficients already include the reduction factor equal to 0.825, accounting for the lack of seismic detailing. If longitudinal bars are lapped in members with plain bars, another coefficient has to be adopted, depending on the lap length ($l_o$) and the shear span ($L_V$). For total chord rotations, it is given by:

$$0.0025(180 + \min(50, l_o/d_{bl}))(1 - l_o/L_V)$$

while for the only plastic part it is:

$$0.0035(60 + \min(50, l_o/d_{bl}))(1 - l_o/L_V)$$

Moreover, shear span in expressions (1) and (2) should be reduced by the lap length $l_o$, assuming that the ultimate condition is controlled by the region right after the end of the lap. The ultimate rotation may also be calculated following an equivalent mechanical approach through the evaluation of the ultimate section curvature, assumed to be constant over the plastic hinge length, which is empirically calibrated. Hence, the ultimate rotational capacity may be evaluated according to [EC8 - Eq. (A.4)]:

$$\theta_{um} = \frac{1}{\gamma_{el}} \left( \theta_y + (\phi_u - \phi_y)L_{pl} \left( 1 - 0.5 \frac{L_{pl}}{L_V} \right) \right)$$

Section curvatures at ultimate and at yielding are calculated based on the first principles, with the constitutive relationships given by Eurocode 2 (CEN, 2004). If the concrete confinement model given in 3.1.9 in Eurocode 2 is assumed, the plastic hinge length is equal to [EC8 - Eq. (A.5)]:

$$l_{pl} = \frac{f_y}{f_{cy}}$$
If the confinement model proposed by Eurocode 8 – part 3 is adopted, better representing the effects of confinement under cyclic loading, the plastic hinge length is given by:

$$L_{pl} = \frac{L}{30} + 0.20h + 0.11 \frac{d_{bl} f_y}{f_c} + 0.17h + 0.24 \frac{d_{bl} f_y}{\sqrt{f_c}}$$

For expressions (7) and (8) no correction factor accounting for the above mentioned deficiencies is given. Therefore, they should only be applied to members with deformed bars, seismically detailed and without lapping of longitudinal bars.

2.2 Mechanical approach: background theory

From a phenomenological standpoint, the plastic hinge region can be identified with the zone of the element where yielding of reinforcement and concrete crushing take place. The plastic hinge length used in the evaluation of the element rotational capacity is, instead, purely conventional. It only represents the length over which ultimate section curvature, assumed to be constant, is integrated, following an equivalent bending approach, to calculate the effective chord rotation including shear and fixed-end rotation contributions to the overall deformability of the member; the curvature is calculated based on Bernoulli’s plane section assumption.

The plastic hinge length cannot be evaluated based on a purely mechanical approach. As a matter of fact, based on section equilibrium conditions and full-interaction hypothesis, in a post-peak phase the curvature should increase only at the base section of the element (“zero length hinge problem”) (Daniell et al., 2008; Haskett et al., 2009). Moreover, a purely mechanical approach, leading to the evaluation of flexural deformability, would not account for other deformation mechanisms such as shear deformability and slippage of reinforcing bars from the connection element. These contributions are not negligible at all. Shear mechanisms may contribute in the overall post-elastic member deformability up to 30% (Fenwick et al., 1993), whilst the end rotation due to the slippage of reinforcing bars may contribute up to 40% (Sezen, 2002).

Therefore, researchers over years have empirically calibrated the plastic hinge length over which theoretical ultimate section curvature is integrated, aiming at achieving the best agreement with experimental values of ultimate chord rotation.

Following this approach, rotational capacity of an element may be expressed as:

$$\theta_u = \theta_y + (\phi_u - \phi_y) L_{pl}$$

where length $L_{pl}$ is made up of three terms, corresponding to different deformation mechanisms:

$$L_{pl} = L_{pl,\text{flex}} + L_{pl,\text{shear}} + L_{pl,\text{slip}}$$

Table 1 reports main formulations that have been proposed over years, starting from the first fundamental work by Baker (Baker et al., 1956). These expressions show that the shear span $L_S$ and the section depth $h$ are the major variables influencing the plastic hinge length, while
the term corresponding to fixed-end rotation is generally proportional to diameter and yielding strength of longitudinal reinforcement bars. First proposed formulations are mainly calibrated based on experimental tests on beam elements, therefore the fixed-end rotation contribution is not clearly evaluated. In recent formulations, calibrated also on column elements, this contribution is clearly represented instead.

Moreover, in (9) the ultimate condition is given in terms of curvature $\phi_u$, depending, based on plane section hypothesis, on steel or concrete failure. Nevertheless, the evaluation of ultimate curvature is not easy or univocal, due to the influence of some aspects as concrete confinement, spalling of the concrete cover or buckling of compressive reinforcing bars. For example, the use of different confinement models may significantly influence the determination of the ultimate curvature, therefore the plastic hinge length can assume very different values.

The plastic hinge formulation proposed in (Panagiotakos et al., 2001) is the most interesting among the expressions presented in literature. It is based on an extensive experimental database, which will be discussed in the next paragraph.

### Table 1. Empirically derived hinge lengths.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Hinge Length ($L_{pl}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Baker et al., 1956)</td>
<td>$k_1k_2k_3(z/d)^{1/4} \cdot d$</td>
</tr>
<tr>
<td>(Mattock, 1964)</td>
<td>$\frac{d}{2} \left[ 1 + \sqrt{\frac{1.14z}{d} - 1} \right] \left( 1 - \frac{q - q_y}{q_b} \sqrt{\frac{d^2}{16.2}} \right)$</td>
</tr>
<tr>
<td>(Corley, 1966)</td>
<td>$\frac{d}{2} + 0.2\frac{z}{\sqrt{d}}$</td>
</tr>
<tr>
<td>(Mattock, 1967)</td>
<td>$\frac{d}{2} + 0.05z$</td>
</tr>
<tr>
<td>(Park, 1982)</td>
<td>$0.4h$</td>
</tr>
<tr>
<td>(Priestley et al., 1987)</td>
<td>$0.08L_v + 6d_b$</td>
</tr>
<tr>
<td>(Paulay et al., 1992)</td>
<td>$0.08L_v + 0.022d_b f_y$</td>
</tr>
<tr>
<td>(Panagiotakos et al., 2001)</td>
<td>$0.12L_v + 0.014d_b f_y$ for cyclic loading</td>
</tr>
<tr>
<td></td>
<td>$0.18L_v + 0.021d_b f_y$ for monotonic loading</td>
</tr>
<tr>
<td>(Fardis, 2007)</td>
<td>$0.09L_v + 0.2h$</td>
</tr>
<tr>
<td></td>
<td>$0.04L_v + 1.2h$ for monotonic loading</td>
</tr>
</tbody>
</table>

The ultimate chord rotation is given by:

$$\theta_u = \frac{\phi_u L_v}{3} + (\phi_u - \phi_y) L_{pl} \left( 1 - 0.5 \frac{L_{pl}}{L_v} \right)$$  \hspace{1cm} (11)

and the plastic hinge length $L_{pl}$ is given as a linear function of shear span $L_v$ (bending contribution) and of the product $(f_y d_{bl})$ (fixed-end contribution):

$$L_{pl} = \alpha L_v + \beta (f_y d_{bl})$$  \hspace{1cm} (12)
Coefficients $\alpha = 0.12$ and $\beta = 0.0014$ are derived from a regression analysis on experimental data from cyclic tests. The ultimate curvature $\phi_u$ is evaluated accounting both for the concrete confinement and for the spalling of the concrete cover. The mean and median of the experimental-to-predicted ratio for expression (11), using (12), are equal to 1.23 and 0.99 respectively, with a Coefficient of Variation (CoV) of 83%.

The last plastic hinge expression proposed by (Fardis, 2007), based on a more extensive experimental database, is depending not on the shear span $L_V$ but also on the height $h$ of the section. Moreover, the fixed-end rotation contribution is evaluated with a separate term:

$$\theta_u = \theta_y + (\theta_{u,\text{slip}} - \theta_{y,\text{slip}}) + (\phi_u - \phi_y)L_{pl} \left(1 - 0.5 \frac{L_{pl}}{L_V}\right)$$

(13)

with:

$$L_{pl} = 0.09L_V + 0.20h$$

(14)

where:

$$\theta_y = \frac{\phi_y L_V}{3} + 0.0013 \left(1 + 1.5 \frac{h}{L_V}\right) + \frac{\phi_y d_{bl} f_y}{8 \sqrt{f_c}}$$

(15)

$$\theta_{y,\text{slip}} = \frac{\phi_y d_{bl} f_y}{8 \sqrt{f_c}}$$

(16)

$$\theta_{u,\text{slip}} = \frac{\phi_u d_{bl} f_y}{16 \sqrt{f_c}}$$

(17)

The use of the illustrated relationships, together with the confinement model showed in the same work, leads to an experimental-to-predicted ratio with mean and median, on a database of 1307 experimental tests, equal to 1.105 and 0.994 respectively, with a CoV of 53.6%.

Expressions (11) and (13), although providing a different evaluation of the fixed-end contribution, present the same control variables of the code expression (6), which directly shows, in the calculation of plastic hinge length, the dependence on all the above mentioned parameters.

### 2.3 Empirical approach: background theory

Formulas for the evaluation of rotational capacity can also be obtained with a totally empirical approach, based on experimental data, with pure numerical regression analyses. Different empirical expressions are proposed in literature (Rossetto, 2002; Zhu et al, 2007); among them, the expression proposed in (Panagiotakos et al., 2001) is certainly based on the most extensive database. Therefore, it is the most representative and it represents a reference for code formulas (CEN, 2005).

This experimental database consists in 633 cyclic tests and 242 monotonic tests on beams, columns and walls, which do not present brittle failure mechanisms. The relationship is a linear regression of the logarithm of $\theta_u$ on the control variables or their logarithms, without
coupling. Only control variables which turn out to be statistically significant for the prediction of \( \theta_u \) are retained. Separate regression analyses for monotonic tests and for cyclic ones are performed. To obtain a more representative experimental database, with particular regard to members with unsymmetric reinforcement well represented in monotonic tests, another regression analysis on all 875 tests is performed, carrying to the following expression:

\[
\theta_u = \alpha_{st} \cdot \alpha_{cyc} \cdot \left( 1 + \frac{\alpha_{sl}}{2.3} \right) \left( 1 - \frac{\alpha_{wall}}{3} \right) (0.20)^y \left[ \frac{\max(0.01;\alpha')}{\max(0.01;\alpha)} \right]^{0.275} \left( \frac{L_V}{h} \right)^{0.45} \left[ \frac{\rho_{apx} f_{yw}}{f_c} \right]^{100}(1.30^{100\rho_d})
\]

where \( \alpha_{cyc} \) is a binary coefficient given equal to 1 for monotonic loading and equal to 0.6 for cyclic loading. The ratio between the experimental ultimate rotation and the numerical value provided by (18) has mean equal to 1.06, median equal to 1.00 and CoV of 47%.

During years, together with the extension of the experimental database, the coefficients in this expression have been slightly modified. The last proposal, given in (Fardis, 2007), is based on 1307 monotonic and cyclic tests:

\[
\theta_u = \alpha_{st} \cdot (1 - 0.43\alpha_{cyc}) \cdot \left( 1 + \frac{\alpha_{sl}}{2} \right) \left( 1 - \frac{3}{8} \alpha_{wall} \right) (0.30)^y \left[ \frac{\max(0.01;\alpha')}{\max(0.01;\alpha)} \right]^{0.225} \left( \frac{L_V}{h} \right)^{0.35} \left[ \frac{\rho_{apx} f_{yw}}{f_c} \right]^{100}(1.25^{100\rho_d})
\]

where \( \alpha_{st} \) is equal to 0.0185 for hot-rolled ductile steel, 0.0115 for heat-treated (tempcore) steel, and 0.0090 for cold-worked steel. The mean value of the ratio between the experimental ultimate rotation and the numerical value provided by (19) is 1.05, the median is equal to 0.995 and the CoV is of 42.8%. The comparison between the coefficients of variation clearly shows the better prediction capacity of (19), given by the growth of experimental knowledge state.

In the same work, a regression analysis for the only plastic part is also presented, which was already proposed in (CEB-FIB Bulletin 24, 2003) based on 1100 experimental tests. The expression is:

\[
\theta_{u}^{pl} = \alpha_{st}^{pl} \cdot (1 - 0.52\alpha_{cyc}) \cdot \left( 1 + \frac{\alpha_{sl}}{1.6} \right) \left( 1 - 0.4\alpha_{wall} \right) (0.25)^y \left[ \frac{\max(0.01;\alpha')}{\max(0.01;\alpha)} \right]^{0.50} f_c^{0.20} \left( \frac{L_V}{h} \right)^{0.35} \left[ \frac{\rho_{apx} f_{yw}}{f_c} \right]^{100}(1.275^{100\rho_d})
\]

The mean value of the ratio between the experimental ultimate rotation and the corresponding numerical prediction is 1.05, the median is equal to 0.995 and the CoV is of 42.7%, against the 47% in the first proposal (see Eq. 18).
Expressions (1) and (2) proposed in EC8 almost perfectly agree with (19) and (20), assuming $\alpha_{\text{cyc}} = 1$ (cyclic loading), $\alpha_{\text{sl}} = 1$ (with slip), $\alpha_{\text{wall}} = 0$ (only beams and columns) and $\alpha_{\text{st}} = \alpha_{\text{pl}}^\text{st}$ equal to 0.0185 (hot-rolled ductile steel).

Consistently with the characteristic of tests included in the experimental database, the proposed expressions for the ultimate rotational capacity should be applied only to members with deformed bars, with seismic detailing and without lapping of longitudinal bars in the vicinity of plastic hinge region, that is, to members which are not representative of existing buildings. Authors define correction coefficients allowing to extend the use of these expressions to members with different characteristics. These coefficients are calibrated to counterbalance the mean error evaluated through the comparison between values from expressions (19) and (20) and results of experimental tests on under-designed members, not included in the original (primary) database. This approach, certainly approximated, is necessary because of the small number of experimental data for these members. Because of the low number of these data, it seems to be allowed to suppose that their inclusion in the database would have not led to any significant change in the regression expression. Moreover, applying the primary expression to members of different typologies, only using a multiplicative coefficient, is the same as postulating that the ultimate rotation depends on the control parameters by the same way, independently on the specific characteristics of considered elements. Nevertheless, the assumed methodology seems to be the only one that can be followed, due to the few experimental data now available for this kinds of elements. A higher reliability can be obtained only by extending the experimental database, so that a wider range of loading conditions and geometrical and mechanical characteristics can be covered. In Table 2 correction coefficients and the extension of the corresponding experimental database used for calibrations are reported.

Table 2. Correction factors for non-detailed members.

<table>
<thead>
<tr>
<th>Element Type</th>
<th>Correction Factor</th>
<th># of Data</th>
<th>Mean - Median CoV</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>w/o seismic detailing and continuous ribbed bars</td>
<td>0.85</td>
<td>27</td>
<td>0.81 - 0.85</td>
<td>42% (Panagiotakos et al., 2002; CEB-FIB Bulletin 24, 2003)</td>
</tr>
<tr>
<td>w/o seismic detailing w/ hooked plain bars and w/ or w/o lap-splicing over plastic-hinge length</td>
<td>0.015 - (10 + \min(40; l_0/d_h))</td>
<td>15</td>
<td>1.07 - 0.975</td>
<td>32% (Fardis, 2006)</td>
</tr>
</tbody>
</table>

2.4 Critical review

The expressions for the ultimate rotational capacity, as clearly shown in the previous paragraphs, are necessarily calibrated on experimental data, due to the complex nature of mechanisms affecting the post-elastic behaviour of reinforced concrete members and their interaction.

Both the approaches presented in literature and in Code are characterized by high values of the coefficient of variation of the experimental-to-predicted capacity ratio. The high CoV affecting expressions (19) and (20) – or (1) and (2) - is not only due to the natural experimental variability, but also to the difficulty in completely modelling with a simple formulation the interaction between the complex phenomena influencing the post-elastic deformation behaviour of reinforced concrete element. Panagiotakos and Fardis in (Panagiotakos et al., 2001), based on the analysis of subgroups of tests, homogenous for geometrical and mechanical characteristics and for loading conditions, quantify the CoV associated with the only natural variability in 12.5%.
The limited prediction capacity of these expressions is also due to impossibility of introducing in the control variables some parameters which certainly affect the rotational capacity. The major among these parameters is the load path, that is the energy dissipated in hysteretic cycles. This aspect has been experimentally investigated by (Pujol et al., 2006), who analyzed the influence of displacement history on the decay of element resistance capacity. The experimental tests show that, given equal the geometrical and mechanical characteristics and the applied axial load (that is, all the input parameters of code and literature regression formulations), it is possible to **predetermine** the value of element chord rotation corresponding to a conventional drop of 20 % of peak resistance, by imposing a given load path (cfr. Figure 1).

![Figure 1. Influence of displacement history on ultimate chord rotation (Pujol et al., 2006).](image)

Panagiotakos and Fardis, in the above mentioned work, try to explicitly account for the effect of cyclic loading by another regression, where the type of loading is evaluated with a variable expressing the equivalent number of inelastic imposed cycles \( \sum |\theta_i| \theta_a \), instead of the coefficient \( \alpha_{\text{cyc}} \). Nevertheless, contrary to expectations, the inclusion of this parameter makes worse the prediction capacity of the formulation. The coefficient of variation (CoV) of the ratio between the experimental and the predicted value, in fact, increases up to 51 %. On the other hand, the usual structural modelling approaches do not allow to introduce the dissipated energy in the control variables.

A critical analysis of expressions (19) and (20), based on mechanical considerations regarding the absence of a direct relationship between the median estimation of the ultimate rotation and some parameters that certainly influence the member capacity, seems to be without foundation. Due to the purely statistical nature of the expression, in fact, the retaining of these variables turns out to be not significant because of their strong correlation with other parameters, already present in the formulation (Panagiotakos et al., 2001).

It’s worth noting that the higher coefficient of variation affecting the hybrid mechanical-empirical formulation (plastic hinge length) with respect to the purely empirical one is probably related to the difficulty in expressing the ultimate rotation as a function of element characteristics based on a statistical regression analysis restrained to a mechanical relationship.
3 DEFORMATION CAPACITY OF RC MEMBERS WITH PLAIN BARS

Plain reinforcing bars have been widely used in the construction of European reinforced concrete buildings. In Italy and in the whole Mediterranean area, their use was widely spread up to 1970s, in north-American countries and in New Zealand construction with plain bars are present until 1950. The high spreading of reinforced concrete buildings with plain bars among existing buildings can be deduced if it is considered that 50 % of Italian existing buildings has been constructed between earliest 1940s and latest 1970s, when reinforced concrete structures with plain bars were the prevailing construction typology.

The correct evaluation of deformation capacity of R.C. elements has to account for the effective bond capacities between reinforcing bars and the surrounding concrete. For members with plain bars, low bond capacities directly influence the three main deformation mechanisms: bending, shear and fixed-end rotation.

As shown by experimental evidence, the scarce capacities of load transfer between the reinforcing bars and the surrounding concrete makes the deformation contribution associated with the fixed-end rotation effect very important. This contribution, in fact, due to the cyclic and post-elastic decay of bond capacities, may represent up to 80-90 % of overall deformability of the element (Verderame et al., 2008a; Verderame et al., 2008b).

Bond capacities also influence the development of cracks along the element. A lower number of wider cracks is observed when bond decreases. This greatly influences both shear and bending deformability, reducing the former and increasing the latter.

Therefore, formulations able to provide a reliable assessment of ultimate deformation capacity of elements with plain bars are of a particular interest for assessment of existing buildings. The ultimate rotational capacities for members with plain bars, according to code, as already shown at paragraph 2.1, is evaluated by applying a correction coefficient to the capacity formulations calibrated on members with deformed bars and seismically detailed. In the following a new calibration of these coefficients, which result to be too conservative, is proposed, based on an experimental database of columns with plain bars extended with recent experimental results from tests executed in the laboratory of the Department of Structural Engineering at the University of Naples “Federico II”, in the research project ReLUIS-DPC 2005-2008 Linea 2.

3.1 Experimental data set

Most of literature data about the experimental behaviour of R.C. elements comes from test executed on members with deformed bars. During last years, the need for a reliable assessment of seismic capacity of existing structures has produced an increasing number of experimental campaigns aimed at the study of behaviour of under-designed elements. This allowed to extend the experimental database for the calibration of correction coefficients applied to the regression relationships for the evaluation of ultimate rotational capacity. Coefficients proposed in (CEN, 2005), reported at 2.1, are calibrated on very few experimental tests. Expression proposed in (Fardis, 2006), reported in Table 2, are, instead, based on 15 experimental tests; 9 of them without lapping \( l_o / d_{bl} = 100 \) and 6 with a lap length \( l_o \) varying between 15, 25 and 40 times the diameter \( d_{bl} \) of longitudinal reinforcing bars. The comparison between the code correction coefficient and the latest one proposed in (Fardis, 2006) shows the considerable conservativeness of the prescription proposed by Eurocode 8.

In recent times, in the Department of Structural Engineering at the University of Naples “Federico II”, a great attention has been addressed to the experimental study of members with
plain bars, both through test aimed at the characterization of bond capacities in cyclic (Verderame et al., 2009a; Verderame et al., 2009b) and post-elastic field (Verderame et al., 2008c) and through tests on real-scale columns elements under monotonic and cyclic loading. The first phase of the experimental activity 6 monotonic test e 6 cyclic ones have been performed, on elements with square section (300x300mm²), for different values of the applied axial load. In this phase particular attention has been addressed to the detail of longitudinal bars, by executing tests on elements without lapping of longitudinal bars at the base of the element and on elements with a lap length l_o equal to 40 times the diameter d_lo of longitudinal bars.

The second phase of the experimental campaign, just finished and still unpublished, is focused on the comparison between rotational capacity and deformation mechanisms of R.C. elements with plain and ribbed bars. In particular, 4+4 tests have been executed on elements equal for the geometry of the transverse section, the geometric ratio of longitudinal and transverse reinforcement, the axial load level and the load path, varying the geometry of the transverse section. The characteristics of tested elements are reported in Table 3, where the drift corresponding to the 20 % decay of the peak resistance is also given.

<table>
<thead>
<tr>
<th>n test</th>
<th>Reference</th>
<th>b [mm]</th>
<th>L_v/h</th>
<th>P/A_f</th>
<th>ρ_l [%]</th>
<th>l_o/d_lo</th>
<th>reinforcement Type</th>
<th>loading</th>
<th>θ_u,exp [20%] [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>300</td>
<td>5.23</td>
<td>0.12</td>
<td>0.8</td>
<td>40</td>
<td>Plain</td>
<td>cyclic</td>
<td>6.23</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>300</td>
<td>5.23</td>
<td>0.24</td>
<td>0.8</td>
<td>40</td>
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</table>

By adding these tests, the database for the evaluation of the correction coefficient applied to the ultimate rotational capacity of elements with plain bars consists of 26 tests, 7 of which monotonic. It’s worth noting that tests (#1,2,3,5,6) were already included in the database used by Fardis for calibrating expressions reported in Table 2 (Fardis, 2006).

### 3.2 Calibration of correction factor

In this paragraph the correction coefficient applied to code expressions for the ultimate rotation of members with plain bars is calibrated, based on experimental data introduced at 3.1.
The correction coefficients will be calibrated according to the methodology already illustrated at 2.3, with regard to the following expression:

$$\theta_u = \theta_y + \theta_{u, pl}$$  \hspace{1cm} (21)

with:

$$\theta_{u, pl} = 0.03 \cdot (1 - 0.52\alpha_{cyc})(0.25)^{0.3} \left[ \max\left(\frac{\mu_1}{\omega} \right) \right]^{0.2} \left( \frac{L_V}{h} \right)^{0.35} \left( \frac{f_{yw}}{f_c} \right)^{25} \left( 1.275 \frac{1000}{d_l} \right)$$  \hspace{1cm} (22)

It is to be noted that a factor accounting for the type of loading ($\alpha_{cyc}$) has been added to the code expression (2) for the plastic part of the ultimate rotation $\theta_{u, pl}$. This assumption is considered to be allowed because of the almost perfect agreement between the code expression and the one proposed in (Fardis, 2007), as already shown at 2.3.

Table 4 reports, for all experimental tests in the database, the ratios between experimental ultimate rotation and the corresponding theoretical value ($\frac{\theta_{u, exp}}{\theta_u}$), according to (22).

<table>
<thead>
<tr>
<th>n test</th>
<th>Reference</th>
<th>loading</th>
<th>$l_u/d_{bl}$</th>
<th>$\frac{\theta_{u, exp}}{\theta_u}$</th>
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The ratio ($\frac{\theta_{u, exp}}{\theta_u}$) for members without lapping of longitudinal bars (conventionally reported as $l_u/d_{bl} = 100$) has mean equal to 0.99 and median equal to 0.87, with a CoV of 37%. Hence, based on the experimental database, it can be deduced that the assessment of the
ultimate rotation with (22) overestimates the median value of rotational capacity by about 13%.

The use of expression (22) for members with lapping of longitudinal bars overestimates even more the experimental rotational capacity. A linear regression performed on the ratio ($\theta_{u,\text{exp}} / \theta_u$) gives the following expression for the correction coefficient:

$$k = 0.020 \min(45, \frac{l_o}{d_{bl}})$$

(23)

This coefficient, applied also to elements without lapping, allows to account for the overestimate of the rotational capacity given by (22); in particular, the rotational capacity evaluated according to (22) is reduced by 10%. The ratio [$\theta_{u,\text{exp}} / (k\theta_u)$], calculates on all tests in the experimental database, has mean equal to 1.10 and median equal to 1.01, with a CoV of 37%.

Figure 2a reports, for each experimental test, both cyclic and monotonic, the ratio between the experimental ultimate rotation and the corresponding theoretical value ($\theta_{u,\text{exp}} / \theta_u$), together with the correction coefficient given by (23), which should be applied to (22).

It is noted that including monotonic tests in the evaluation of the correction factor (k) is the same as postulating that the reducing of rotational capacity due to cyclic loading, evaluated in (22) by the coefficient $(1 - 0.52\alpha_{\text{cy}})$, is, on average, not depending on bond capacities. As a matter of fact, this coefficient, as previously illustrated, is calibrated on a database made up of members with deformed bars; therefore, the evaluation of the correction coefficient (k) has been executed supposing that the reduction given by $(1 - 0.52\alpha_{\text{cy}})$ can also be extended to members with plain bars.

Nevertheless, from a theoretical standpoint, a member with deformed bars, given equal the geometrical and mechanical characteristics, should show a higher cyclic degradation with respect to a member with plain bars, because of the micro-cracking of the concrete surrounding the reinforcing bar due to the higher bond performances, which emphasizes the strength degradation of concrete alternatively in compression and in tension.

Figure 2. Proposed correction factor: (a) cyclic and monotonic tests, (b) only cyclic tests.
However, due to the uncertainties related to the inclusion of monotonic tests, the correction coefficient will now be calibrated based on the only cyclic tests. For these tests, the ratio \( \left( \frac{\theta_{u,\text{exp}}}{\theta_u} \right) \) for elements without lapping of longitudinal bars has mean equal to 1.02 and median equal to 0.98, with a CoV of 39%. Therefore, based on the experimental tests, expression (22) shows a very good agreement with the cyclic rotational capacity of elements with plain bars without lapping of longitudinal reinforcement.

A linear regression performed on the ratio \( \left( \frac{\theta_{u,\text{exp}}}{\theta_u} \right) \), for elements with lapping of longitudinal reinforcement, gives the following expression for the correction coefficient:

\[
k = 0.020 \min(50, l_o / d_{bl})
\]  
(24)

Figure 2b reports, for each cyclic experimental result, the ratio between the experimental ultimate rotation and the corresponding theoretical value \( \left( \frac{\theta_{u,\text{exp}}}{\theta_u} \right) \), together with the correction coefficient given by (24), applied to (22).

### 3.3 Discussion of results

The extension of the experimental database allowed to re-calibrate the correction coefficients applied to the assessment of the ultimate rotational capacity of elements with plain bars, with or without lapping of longitudinal reinforcement.

The choice between the correction coefficient calibrated only on cyclic tests or on the whole experimental database is not easy. The numerical results, in fact, are rather different. Although the inclusion of monotonic tests in the database is affected by the previously highlighted uncertainties, it is to be noted that the consideration of the only cyclic tests would imply a further reduction in the extension of the experimental database, which is already limited.

However, both the expressions of correction coefficients proposed in the present work highlight the conservativeness of EC8 proposal, which is based on very few experimental tests. Moreover, EC8 assumes that, when lapping of longitudinal reinforcement is present, the ultimate condition is controlled by the region right after the end of the lap, so that the shear span and, therefore, the rotational capacity are further reduced, but this assumption is not confirmed by the experimental results. The highest plastic demand, in fact, always concentrates at the base section of the element.

Despite the difficulties in the choice of the most reliable expression for the correction coefficient, recent experimental results clearly highlight the higher rotational capacity of members with plain bars with respect to ones with deformed bars, equal for structural characteristics and details. As a matter of fact, the comparison between the ultimate rotations of the elements from the second phase of the experimental campaign, briefly illustrated at 3.1, highlight that the capacity of members with plain bars are higher, on average, by 35% compared with the corresponding members with deformed bars (see Table 3).

From a mechanical standpoint, the higher ultimate rotational capacity of columns with plain bars may be explained with the comparison between two opposite mechanisms: the increase of deformability caused by the fixed-end rotation mechanism, particularly exalted due to the low bond capacities; on the other hand, the higher degradation of global resistance due to the increase of deformation demand on concrete in compression, localized at the base of the element and associated with the concentrated rotation (“rocking effect”). According to experimental evidence, the former seems to prevail on the latter, leading to an overall increase of ultimate rotational capacity compared with members with higher bond capacities.
4 CONCLUSIONS

In this work, the theoretical background to code formulas for the assessment of ultimate rotational capacity of reinforced concrete members has been briefly presented. Most recent literature contribution, together with advantages and deficiencies of the approaches to the calibration of this relationships, have been illustrated.

Special attention has been addressed to the calibration of correction coefficients used for the assessment of ultimate rotational capacity of under-designed elements, with emphasis on members with plain bars.

Main conclusions drawn from this work are:

- The evaluation of post-elastic deformation capacity of r.c. elements may only be based on experimental data; any mechanical approach would not allow to evaluate accurately the complex interaction phenomena influencing the deformability of the element.
- The reliability of regression expressions proposed in literature, some of which have been adopted in code, is a direct result of the extension and the correct sorting of the database.
- The estimate of rotational capacity of under-designed elements is strongly influenced by the low number of experimental data related to this typologies.
- Recent experimental tests on columns with plain bars, executed at the University of Naples (DIST), allow to extend significantly the database used for the calibration of correction coefficients applied to the assessment of these elements, with or without lapping of longitudinal reinforcement.
- The re-calibration of correction coefficients, even within the limits of the adopted methodology, has allowed to highlight the excessive conservativeness of current code prescriptions for elements with plain bars; this is confirmed by the experimental evidence, showing that the ultimate rotation of members with plain bars is higher compared with members with deformed bars, on average, by 35 %, given equal the structural characteristics and details.

5 ACKNOWLEDGEMENTS

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6 REFERENCES


