A DESIGN METHOD FOR STEEL FRAMES EQUIPPED WITH BUCKLING RESTRAINED BRACES

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INTRODUCTION

Buckling-restrained braced frames were introduced to enhance the compressive capacity of braces while not affecting their stronger tensile capacity, hence producing a symmetric hysteretic response [1]. A buckling restrained brace (BRB) usually consists of a core steel brace encased in a steel tube that may be filled with concrete or grout. BRBs have been used extensively for seismic applications in Japan and United States due to their simple and efficient behaviour, as testified by several applications [1,2] and by the inclusion in code recommendations such as the AISC seismic provisions [3]. The introduction of BRB members undoubtedly represents a major advancement compared to conventional braces in terms of cyclic inelastic deformation capacity and reduction of design forces. However, buckling restrained braced frames may undergo large inelastic storey drifts without the ability to distribute the ductility demand over the height of multi-storey structures due to possible localizations of inelastic deformations. This latter aspect deserves particular attention due to the bracing system characteristics, i.e., statically determinate structural configuration and limited BRB hardening, especially in steel frames with beams connected to columns by means of pinned joints as often happens in Europe. As a result, the frame global ductility is strongly dependent on the distribution of BRB strength and stiffness at each storey level.
In this paper a single degree of freedom (SDOF) based design method for BRB steel frames is examined. The bracing system is modelled as a continuum cantilever beam where BRBs are associated to the shear stiffness and columns are associated to the flexural stiffness. This continuum model allows a more clear identification of the parameters influencing the structural behaviour and a more simple definition of the design procedure as compared to discrete models. Closed form solutions can be obtained for cantilever beams with uniformly distributed mass, resulting in simple analytical expressions that can be adopted for the design of structures with regular mass distribution over the height. An optimal solution for the SDOF system is achieved similarly to other methods based on the same type of approach [4][5]. However the actual dynamic nonlinear response can significantly be influenced by higher vibration modes and by deformation localization at some floor levels. Thus preliminary results of numerical simulated response analyses are illustrated in order to highlight advantages and limitations of the presented design procedure paying particular attention to the differences between linear static analysis based on the first vibration mode and nonlinear dynamic analysis.

1 PROPOSED DESIGN METHOD

1.1 Dynamics of shear-deformable beams

The bracing system behaviour is described by a continuous model consisting in a cantilever beam including flexural and shear deformability. The balance equations and relevant boundary conditions of this continuous model are:

$$\mu\ddot{v} - [A(u'+\psi)]' = q \qquad A(u'+\psi)\big|_L = 0 \qquad u\big|_0 = 0 \tag{1}$$

$$I_\mu\ddot{\psi} - (J\psi')' + A(u'+\psi) = 0 \qquad J\psi'\big|_L = 0 \qquad \psi\big|_0 = 0 \tag{2}$$

where the functions $u(\zeta;t),\psi(\zeta;t):[0,L]\times[0,\infty)\mapsto\Re$ are the cross section transverse displacements and rotations respectively, L is the total height, $q(\zeta;t)$ is the time dependent load distribution, $A(\zeta)$ and $J(\zeta)$ are the distribution of flexural and shear stiffness respectively, $\mu(\zeta)$ is the mass per unit length, $I_\mu(\zeta)$ is the rotation inertia, a prime denotes one differentiation with respect to ζ and a superposed dot denotes one differentiation with respect to time t [6]. The strain field is described by the shear deformation $\gamma = u'+\psi$ and by the curvature $\theta=\psi'$. The solution is defined once the initial conditions are assigned. In the sequel the rotation inertia is neglected.
The proposed design method aims at finding a stiffness distribution giving uniform shear deformation $\bar{\gamma}$ and uniform curvature $\bar{\theta}$ along the cantilever in the first vibration mode. The shear deformation is related to the deformation of the diagonal braces while the curvature is related to the deformation of the columns. Thus the uniform strain condition leads to simultaneous BRB yielding if the response is dominated by the first vibration mode and an adequate over strength is guaranteed for the columns. Starting from the assumed uniform strain field, it is possible to evaluate the functions $v(\zeta)$ and $\varphi(\zeta)$ describing the transverse displacement and rotation of the beam in the first vibration mode of circular frequency ω, by using the beam compatibility equations and the relevant kinematic boundary conditions:

$$v'+\varphi=\bar{\gamma} \qquad v_i|_0=0 \tag{3}$$

$$\varphi'=\bar{\theta} \qquad \varphi_i|_0=0 \tag{4}$$

The displacement and rotation functions describing the first vibration mode are obtained by integration:

$$v=\bar{\gamma}\zeta-\frac{1}{2}\bar{\theta}\zeta^2 \tag{5}$$

$$\varphi=\bar{\theta}\zeta \tag{6}$$

From the modal shape it is possible to compute the coefficient m^* and the modal participation factor Γ, normalized with respect to the cantilever horizontal displacement of the top end:

$$m^*=\frac{1}{v|_L}\int_0^L\mu(\zeta)v(\zeta)d\zeta \qquad \Gamma=\frac{v|_L^2\,m^*}{\int_0^L\mu(\zeta)v^2(\zeta)d\zeta} \tag{7,8}$$

Thus the stiffness distributions $A(\zeta)$ and $J(\zeta)$ are determined from the differential balance equations of the first vibration mode (inverse problem) deduced substituting Eqs. (3-4) and the relations $u=ve^{i\omega t}, \psi=\varphi e^{i\omega t}$ in Eqs. (1) and (2):

$$-\mu\omega^2 v-A'\bar{\gamma}=0 \qquad A|_L=0 \tag{9}$$

$$-J'\bar{\theta}+A\bar{\gamma}=0 \qquad J|_L=0 \tag{10}$$

The solution of this homogeneous differential system is defined except for a scale factor assumed to be the area of the base section $A_0=A(0)$. The problem is solved taking as unknowns the functions $a(\zeta)=A(\zeta)/A_0$ and $j(\zeta)=J(\zeta)/A_0$. The function a is evaluated by integrating Eq. (9); afterward the function j is obtained from Eq. (10). The circular frequency is defined except for a scale factor and can be given as a function of the circular frequency $\omega_{(1)}^2$ corresponding to a shear stiffness for a unitary base area:

$$\omega^2 = A_0 \frac{\overline{\theta}^2 \int_0^L j d\zeta + \overline{\gamma}^2 \int_0^L a d\zeta}{\int_0^L \mu v^2 d\zeta} = A_0 \omega_{(1)}^2 \tag{11}$$

In the case of uniform mass distribution the following analytical expressions are obtained:

$$a = \frac{\left(1-(\zeta/L)^2\right) - \frac{\beta}{3}\left(1-(\zeta/L)^3\right)}{1-\frac{\beta}{3}} \tag{12}$$

$$j = \frac{L^2}{\frac{\beta}{2}\left(1-\frac{\beta}{3}\right)}\left[\left(-\frac{1}{3}+\frac{1}{2}\zeta/L-\frac{1}{6}(\zeta/L)^3\right)-\beta\left(-\frac{1}{8}+\frac{1}{6}\zeta/L-\frac{1}{24}(\zeta/L)^4\right)\right] \tag{13}$$

where a dimensionless parameter $\beta = \overline{\theta}L/\overline{\gamma}$ is introduced. From the above displacements and stiffness analytical expressions the characteristic parameters m^*, Γ, $\omega_{(1)}$ can be derived.

1.2 Base area design

It is assumed that: (i) the cantilever has a linear elastic – perfect plastic shear behaviour; (ii) the yield shear $V_y = A\overline{\gamma}$ is attained for the shear deformation $\overline{\gamma}$; (iii) the ultimate shear strain is given by $\gamma_u = \mu_d \overline{\gamma}$ where μ_d is the BRB assigned design ductility. The bracing system ductility relevant to the first vibration mode (μ_s) can be deduced by subdividing the displacement of the cantilever top end (point of control) in the flexural contribution $v_c|_L = -\overline{\theta}L^2/2$ and in the shear contribution $v_d|_L = \overline{\gamma}L$ (Eq. 5) and can be put in the form:

$$\mu_s = \frac{v_c|_L + \mu_d v_d|_L}{v_c|_L + v_d|_L} \tag{14}$$

The base area A_0 is designed comparing the capacity of the bracing system to the seismic demand. The capacity may be given by the maximum acceleration achievable in the equivalent simple oscillator and it is directly proportional to the base section area A_0:

$$C(A_0) = \frac{A_0 \overline{\gamma}}{m^* \Gamma} \tag{15}$$

The demand is given by the design spectra of pseudo acceleration $S_a(\omega, \mu_s)$ for the simple oscillator with ductility μ_s and depends on A_0 since $\omega = \sqrt{A_0}\omega_{(1)}$:

$$D(A_0) = S_a\left(\sqrt{A_0}\omega_{(1)}, \mu_s\right) \tag{16}$$

Thus A_0 can be deduced by the equality:

$$C(A_0) = D(A_0) \tag{17}$$

Otherwise it is possible to evaluate the capacity by means of the maximum displacement $C = v|_L$ (scale factor independent) and the demand from the displacement design spectra $D(A_0) = S_d\left(\sqrt{A_0}\omega_{(1)}, \mu_s\right)$ dependent upon A_0 by means of the circular frequency.

2. APPLICATION TO THE DESIGN OF V-BRACING SYSYEMS

2.1 Extension to bracing systems with concentrated masses

The procedure proposed may be adopted also in the case of bracing systems with masses concentrated at floor levels, once the typology of bracing system is chosen. For example, in the case of a V-bracing system with base b, interstorey height h and number of floors p, the modal shape is known once the strains of the diagonal braces ($\bar{\varepsilon}_d$) and the columns ($\bar{\varepsilon}_c$) are assigned. In this the following values of $\bar{\gamma}$ and $\bar{\theta}$ are obtained

$$\bar{\gamma} = 2\bar{\varepsilon}_d L_d^2 / bh \tag{18}$$

$$\bar{\theta} = -2\bar{\varepsilon}_c / b \tag{19}$$

where $L_d = \sqrt{h^2 + (b/2)^2}$.

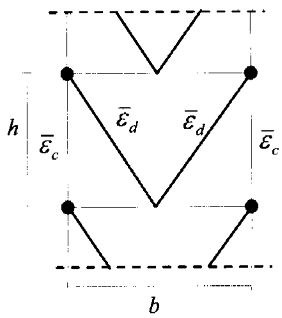

Fig. 1. Geometry of the buckling restrained V- bracing system considered.

The storey displacements v_{di} and v_{ci} due to the diagonal brace deformation and to the column deformation respectively are given by

$$v_{di} = i\bar{\gamma}h \tag{20}$$

$$v_{ci} = v_{ci-1} - i\bar{\theta}h^2 \tag{21}$$

($i = 1 \ldots p$) whereas the storey rotations are

$$\varphi_i = i\bar{\theta}h \tag{22}$$

As previously showed for the continuous case, the normalized shear and flexural stiffness a_i and j_i may be determined, by imposing at each floor the translational and rotational equilibrium equations. The subsequent recursive equations are obtained:

$$a_i = a_{i+1} + m_i v_i / \lambda \tag{23}$$

$$j_i = j_{i+1} + a_i \bar{\gamma} h / \bar{\theta} \tag{24}$$

where $\lambda = \sum_k m_k v_k$ and a_1=1.0 j_1=1.0. From Eqs. (23) and (24) the areas a_{di} of the diagonal braces and the areas a_{ci} of the columns, normalized with respect to the parameter A_0, are obtained:

$$a_{di} = 2a_i L_d^3 / Ehb^2 \tag{25}$$

$$a_{ci} = j_i / 2Eb^2 \tag{26}$$

The circular frequency $\omega_{(1)}$, the modal participation factor Γ and the factor m^* are:

$$\omega_{(1)}^2 = \frac{\bar{\theta}^2 \sum_k j_k + \bar{\gamma}^2 \sum_k a_k}{\sum_k m_k v_k^2} \tag{27}$$

$$\Gamma = \frac{v_p^2 m^*}{\sum_k m_k v_k^2} \tag{28}$$

$$m^* = \frac{1}{v_p} \sum_k m_k v_k \tag{29}$$

2.2 Numerical example

In this section numerically simulated response results for a V-bracing system of six floors, with b = 5 m, h = 3.6 m, floor mass equal to 47 kNs^2/m at the first five levels and equal to 23.5 kNs^2/m at the top floor, are shown. The assumed design strain of the diagonal braces is $\varepsilon_d = 0.00131$, whereas for the columns a lower value equal to $\varepsilon_c = 0.000436$ is chosen in order to guarantee an adequate overstrength of the columns.

The elastic spectrum given by the Italian seismic code [7] for ground types B,C,E with a peak ground acceleration equal to $a_g = 0.3g$ is considered. The design pseudo-acceleration spectrum is obtained by reducing the elastic spectrum by a factor R_μ according to the following equation [4]:

$$R_\mu = (\mu_s - 1)\frac{T}{T_c} + 1 \quad \text{if} \quad T < T_c \tag{30a}$$

$$R_\mu = \mu_s \quad \text{if} \quad T > T_c \tag{30b}$$

with T_c is defined in [7]. For a BRB design ductility equal to $\mu_d = 5$, the proposed design procedure gives the following values of areas (in mm^2) for the diagonal braces and columns respectively: $a_d = [432; 413; 372; 305; 210; 82]$ and $a_c = [4500; 3400; 2400; 1500; 700; 200]$.

Both nonlinear static analysis and nonlinear dynamic analysis (time-history) of the designed bracing system were performed with the finite element structural analysis program SAP 2000 (advanced version 10.1.1). The Bouc-Wen material model with low hardening stiffness (post yield stiffness ratio = 0.02) was adopted for the BRBs.

The nonlinear static push-over analysis, carried out by considering a force distribution correspondent to the first vibration mode of the system, showed that: (i) all BRBs reach yielding and ultimate displacements for the same load multiplier; (ii) the displacement of the point of control (last floor) coincides with the analytically predicted displacement $v_c|_L + \mu_d v_d|_L$.

The results obtained by the nonlinear static analysis are compared with those obtained from nonlinear dynamic analysis of the system subjected to seven artificial accelerograms compatible with the elastic pseudo-acceleration spectrum used in the design procedure. This comparison is illustrated in order to evaluate the influence of higher vibration modes in the seismic response of the structure. Values of the design BRB ductility from $\mu_d = 5$ to $\mu_d = 15$ are considered. In Fig. 2 the maximum peak ground acceleration resisted by the bracing system as deduced from the nonlinear static analysis (continuous line) is compared to the average values obtained from the seven nonlinear dynamic analyses (dashed line). It is observed that the nonlinear dynamic analysis leads to smaller values of the acceleration, primarily for small values of μ_d. Fig. 3 depicts the distributions at each level of the BRB maximum displacements for three values of design BRB ductility (envelope of the maximum displacements for each BRB in the seven nonlinear dynamic analyses). All BRBs attain the yield displacement (i.e., $\varepsilon_d L_d$ = 5.7mm), but in the case of $\mu_d = 5$ the plastic deformation is concentrated at the top floor, whereas for larger ductility values (μ_d =10 and μ_d =15), the first and the last floors show larger plastic deformations.

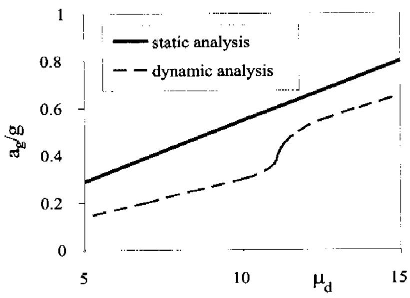

Fig. 2. Maximum peak ground acceleration as a function of BRB ductility: comparison between static and dynamic analyses.

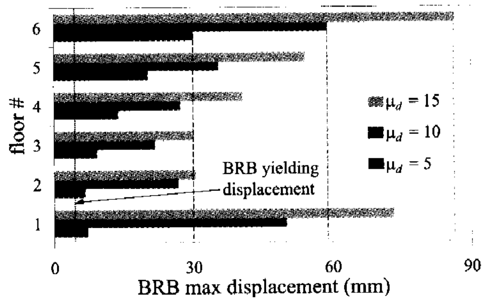

Fig. 3. Distribution of BRB maximum displacements for different values of BRB ductility (nonlinear dynamic analyses).

3 CONCLUSIONS

In this paper a design method for steel frames equipped with BRBs is illustrated. The proposed method aims to obtain an uniform yielding of all BRBs, thus avoiding concentration of plastic deformation at some storey level. This is a critical point in the design since BRBs usually have a very low hardening in post-elastic response. An optimal distribution of column stiffness and of brace stiffness and strength is defined from the free vibration equations of a shear-deformable cantilever. Closed form solutions, useful in preliminary design, are given. Being the proposed method based on the first vibration mode, dynamic analysis results are discussed to evaluate the reduction of performance in the nonlinear range deriving from higher vibration modes. A critical situation in which beams are pinned to columns is considered to highlight the risk of strain localization. In all numerical simulations, BRBs (with ductility spanning from 5 to 15) widely exploited the plastic range and these preliminary investigations furnished satisfactory results.

4 ACKNOWLEDGMENTS

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