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DYNAMIC SYSTEMS WITH HIGH DAMPING RUBBER: NONLINEAR BEHAVIOUR AND LINEAR APPROXIMATION

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ABSTRACT

High Damping Rubber is widely used in seismic engineering and, more generally, in the passive control of vibrations. Its constitutive behaviour is quite complex and is not simply non-linear with respect to strain but also shows a transient response during which material properties change (Mullins effect). A number of recent works were dedicated to analyzing and modelling material behaviour. The present work intends to study the consequences of such non-linear behaviour in the dynamic response of a system where the restoring force is provided by dissipative devices based on HDR (structural system with dissipative bracings and isolated systems). Analyses under harmonic forces and impulsive excitations were carried out in order to characterize the stable and transient responses separately.
Two different linear equivalent systems are deduced from the nonlinear analyses in order to estimate the upper and lower bounds for the dynamic response.
The responses provided by linear and nonlinear models are then used to study the response of the system under seismic excitation in order to compare solutions and evaluate the ability of linear models in furnishing upper and lower bound of maximum displacements and forces.

1. INTRODUCTION

Rubber is a material characterized by shear stiffness many times smaller than bulk stiffness and exhibits large strain without failure and damage. A filler, generally carbon black, is added to the natural rubber in order to improve various mechanical properties such as the dissipation capacity. The damping properties of the material may be particularly beneficial in many situations and it is used in many industrial and engineering applications. In the field of seismic engineering rubber with enhanced dissipating properties, usually known as High Damping Rubber (HDR), is extensively used in bearings for the seismic isolation of bridges or buildings (Grant et al. 2005). It is also used in dissipating devices, generally connected to the structure by means of braces, in order to increase stiffness and energy dissipation capacity. This makes it possible to control the dynamic structural response, reducing lateral displacements in the case of small tremors and damages in the case of strong motions (Fuller et al. 2000, Bartera and Giacchetti 2004, Dall'Asta et al. 2006). The presence of filler, however, entails some difficulty in the use of this material, because the behaviour becomes strongly non-linear and both the stiffness and damping properties vary with the amplitude of strain and depend on strain rate. Furthermore, the response of the material is process-dependent and shows a transient behaviour in which stiffness and damping change remarkably. The phenomenon, usually known as the “Mullins effect”, is a consequence of the damage of the microstructure that occurs during the process (Govinndjee and Simo 1991 and 1992-a, Dorfmann and Ogden 2003). Recent studies (Govinndjee and Simo, 1992-b) show that the transient response is related to the maximum shear strain attained by the material and is influenced by the strain rate. The initial properties of the material are however recovered in a sufficiently short period and this aspect of the problem cannot be neglected in structures undergoing seismic actions (Grant et al. 2005).

In scientific literature, there are several papers attempted to describe and model HDR behaviour. Most of the experimental tests and models proposed however do not concern the seismic behaviour of rubber-based devices (cyclic shear strains), and consider the tension-compression behaviour under loading-unloading paths (Lion 1997, Haupt and Sedlan 2001). Papers that aim at describing the seismic behaviour of HDR devices usually propose constitutive laws based on simple models which are modified in order to take into account the strongly nonlinear strain-dependence and the strain rate-dependence. Only in a few works the process-dependent behaviour is modelled with different approximation levels.

In particular Kikuci and Aiken 1997 present a strain-rate independent elasto-plastic model, with the addition of elastic contributions in order to take into account the strain amplitude dependence and the difference between the first and successive cycles. In Tsai et al 2003 the authors use a modified version of the Bouc-Wen model, adding a linear viscous term. In both cases the dependence on the load history is neglected. An attempt to describe this behaviour may be found in Hwang et al. 2002, but the model proposed is based on separate parameter identification procedures for different load paths (different frequencies, different temperatures, transient and stable behaviours). In addition the results furnished are satisfactory in the case of sinusoidal loads only (Grant et al. 2005). Finally in Yoshida et al., 2004 the authors introduce a damage parameter in order to simulate the degrading of the elastic part, whereas the hysteretic contribution is elasto-plastic and consequently strain-rate and process independent. A test program to fully investigate the pure shear behaviour in a range of strain and strain rates to mitigate seismic effects is described in (Dall’Asta and Ragni 2006). The same paper also proposes an analytical model based on a rheological, thermodynamically compatible, approach, in which internal variables were introduced to describe the inelastic phenomena and the damaging effects.

Previous models are usually quite complex and simplified models, capable of furnishing satisfactory results in the seismic analysis of structures with dissipation devices, may be useful in practical design. In this regard some indications are given by technical codes. In particular linear spring-dashpot models are suggested by some technical codes (OPCM 3431, PrEN 1998-1, FEMA 356) by introducing the concepts of equivalent stiffness and equivalent damping coefficient. Elasto-plastic models are also proposed in some cases (FEMA 356), but these models do not ensure a more accurate description of the structural behaviour and moreover require a non linear analysis. The influence of nonlinear strain-amplitude dependence is taken into account by suggesting iterative procedures where the linear parameters are updated through to displacement converge. In addition the codes recommend considering the influence of the frequency (in a range around the design period), the influence of the temperature and the ageing phenomena by adopting the most unfavourable parameters. Practical codes usually do not furnish clear indications on the transient behaviour of the rubber. The OPCM 3431 code for example recommends determining the linear equivalent parameters considering the third cycle so that the Mullins effect is

neglected. On the other hand, the FEMA 356 code suggests performing multiple analyses with different models, related to transient and stable behaviours, in order to obtain upper and lower bounds for the dynamic response. No accurate indications are however given on the procedure to follow to define the simplified models and on the error produced when using these models. Some studies were carried out in Hwang and Ku 1997 and Hwang and Wang 1998 but these neglected the transient behaviour of the rubber.
The scope of this paper is to study the dynamic behaviour of the non linear system and to propose a criterion for defining the linear models equivalent to the non linear system. Equivalence conditions, which may be used in different situations, are chosen. In particular two reference motions are considered: the first is related to the stable behaviour at resonance conditions (under sinusoidal excitations), whereas the second concerns the transient system response subjected to an impulsive excitation, which is strongly influenced by the Mullins effect. The criterion is applied to three different systems, showing a maximum response in a range period from 0.5s to 2.0s., subjected to shear strain values up to 200%. Results may be of interest for structural systems with different stiffness, from frames with dissipating braces to isolated structures while the strain range considered covers maximum values usually adopted in the design. Numerical results are obtained by adopting the rheological, thermodynamically compatible, model proposed by the authors. In the case of seismic input it may be observed that the two equivalent models, defined according the criterion proposed, furnish the extreme values of forces and displacements with acceptable approximation levels.

2. DYNAMICAL SYSTEMS

2.1 Nonlinear

The S-DoF (Single-Degree of Freedom) dynamical system considered consists of a mass m and a dissipating device, based on HDR subjected to shear strain, which furnishes the restoring force.
The constitutive behaviour of rubber is described by means of the model proposed in (Dall'Asta, Ragni, 2006). This is a thermodynamically consistent rheological model in which the free energy per unit volume $\varphi_d(\gamma;\alpha_i)$ is expressed by means of the *material state* (γ,ξ_i) consisting of the shear strain and a set of internal variables α_i describing the inelastic phenomena related to Mullins effect and viscosity. The shear stress can be deduced by the derivative of the free energy,

$$\tau_d = \frac{\partial \varphi_d}{\partial \gamma} \tag{1}$$

and the evolution of the internal variables is furnished by

$$\dot{\alpha}_i = g(\gamma, \alpha_i, \eta) \tag{2}$$

once the state and the *process* $\eta = \dot{\gamma}$ are known (superposed dot denotes time derivative). The dissipated power per unit volume w_d may be obtained from the derivative with respect to the internal variables (repeated indexes denote summation)

$$w_d = \frac{\partial \varphi_d}{\partial \alpha_i} \dot{\alpha}_i \tag{3}$$

The *state* of the dynamical system is consequently described by the vector $\boldsymbol{x} = [u, v; \alpha_i]$ where u and v are the displacement and the velocity of the mass. Rubber strain $\gamma = \beta u / h$ and strain rate $\eta = \beta v / h$ are proportional to displacement and velocity and may be related by introducing the geometric parameter β depending on the type of connection between the mass and the device. The parameter h is the thickness of the rubber layer.

The restoring force per unit mass f_d may be expressed in the form

$$f_d = \frac{\beta A}{m} \tau_d \tag{4}$$

where A is the area of the *HDR* layer in the device. The evolution law of the dynamic system may be written as follows

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{\alpha}_i \end{bmatrix} = \begin{bmatrix} v \\ -f_d(u, \alpha_i; v) + f_e \\ g_i(u, \alpha_i; v) \end{bmatrix} = \boldsymbol{A}(\boldsymbol{x}) \tag{5}$$

where f_e is the external force per unit mass and the constitutive laws are expressed by means of u and v instead of γ and η.

2.2 Liner equivalent system

In design it is often useful to approach the dynamic problem using simplified linear models that permit easier analyses and a simpler interpretation of results. In the system considered only the relationship between the restoring force and the mass displacement is non-linear so that the search for a simplified model may be reduced to the device only.

Approximation by a linear system consisting of a spring and a dashpot disposed in parallel is considered. This system is fully defined once two parameters are assigned. Hereinafter it will be described by the spring stiffness per unit mass k^L and the dashpot dissipation properties exhibited in the stable response under a sinusoidal deformation history with a circular frequency $\omega^L = \sqrt{k}$. These dissipation properties may be measured by means of the viscous damping coefficient ξ^L defined as the ratio between the average energy dissipated per unit of radiant and the maximum strain energy. The dashpot constant c^L may consequently be obtained by the relation

$$c^L = 2\xi^L \omega^L \tag{6}$$

The restoring force of the linear system is consequently

$$f_d^L = k^L u + 2\xi^L \omega^L v\,. \tag{7}$$

The evolution law of the linear system state $\boldsymbol{x}^L = [u, v]$ has the following form:

$$\dot{\mathbf{x}}^L = \begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ -f_d^L(u,v) + f_e \end{bmatrix} \tag{8}$$

A precise definition of the "dynamic situation", where the equivalence is required, is necessary to define a linear dynamic system that is "equivalent" to a non-linear system. Accordingly, two motions, related to the linear and non-linear systems respectively, must be selected, following from similar external inputs. There must be two equivalence conditions, as a result of the two parameters describing the linear systems, and these must require that the particular quantities observed during the two motions are equal. It is evident that different choices may be made in selecting the dynamic situations and equivalence conditions.

The non-linear system considered shows notable differences between the responses exhibited at the initial stage of motion, influenced by the Mullins effect, and the successive stage, when internal damage reduces stiffness and the dissipative properties. Two different linear models are consequently determined in order to describe the two limit situations. The former model intends to describe the stable behaviour of the material at the assigned strain and strain rate, once Mullins effect is over. This considers the post-transient stable response under sinusoidal external excitation. The maximum periodic motions observed in linear and non-linear system were selected as "dynamic situations". The second model intends to describe the part of the motion strongly influenced by the Mullins effect and considers the initial response under an impulsive excitation. In order to obtain coherent linear approximations, equivalence conditions applicable to different motions are defined.

3. HARMONIC RESPONSE

In this section, the stable response of the non-linear system subjected to sinusoidal external forces is described and successively adopted to define equivalent linear models at different levels of strain.

In the tested range of external actions the system shows a transient response and attains a stable behaviour after a certain number of cycles. The response related to an external action with period T is considered to be stable at the instant t when the difference between the state history observed in the last period and the state history observed in the previous period is sufficiently small. It should be also observed that the dynamical system analyzed exhibits a process-dependent response. In this case, contrary to elastic systems, the stable response obtained depends on the initial conditions. Since the aim is to characterize the response under seismic events acting on the system where the state variables are initially zero, the analyses are performed by assuming $\boldsymbol{x} = \boldsymbol{0}$ for $t = 0$. The external force has the expression

$$f_e(t) = f_0 \sin(2\pi t / T) \tag{9}$$

where f_0 is the amplitude of the force per unit mass and T is its period. Three cases corresponding to three values of stiffness are considered in the analyses. The intermediate case *b* was obtained by a rubber layer with an area *A*=78200 mm^2 and a thickness *h*=10*mm*; a mass of $10^5\,kg$ was considered. The other cases *a* and *c* were obtained with a device area four times larger and smaller, respectively. Such values were chosen in order to obtain dynamical systems that show the displacement peak for external force periods of about 0.5s (case *a*), 1.0s (case *b*) and 2.0s (case *c*). The chosen stiffness values make it possible to study

different situations referable to different structural systems where devices are introduced in order to reduce seismic effects, like frames with dissipating bracings (T=0.5s-1.0s) or isolated structures (T=1.0s-2.0s). For each case, it was assumed $\beta = 1$ and the maximum intensity of the external force f_0 was calibrated to provide maximum values of shear strain ($\gamma = u/h$) equal to 2.0, 1.5 and 1.0, which are the usual maximum strain values used in design .
Figure 1 reports the results for case b. It describes the maximum values of displacements and restoring forces per unit mass observed in the nonlinear system (solid line) together with the results of the linear approximation (dashed line). The diagrams are in a non-dimensional format and were obtained by dividing displacements, forces and periods by reference values defined with the criteria indicated below. Displacement u_{ref} and the reference force f_{dref} , the value of the maximum displacement and the value of the maximum device force attained with the external force f_{0m} are assumed as reference. Since the reference period T_{ref} is assumed to be the period at which the maximum values of the response are reached. Similar results were observed in the other cases a and c.
By observing the nonlinear results it is evident that the main displacement peak occurs at about T=T_{ref} only when the maximum displacement is u_{ref} (the shear strain is γ=2). For the other curves, related to the different force intensity values, the system is stiffer and the periods at which the main peaks occur are remarkably smaller, as is usual for softening systems. Other non linear phenomena may be observed, such as the non proportional relation between external force and maximum displacement and the secondary peak exhibited for periods about 1.8 times those of the main peak.

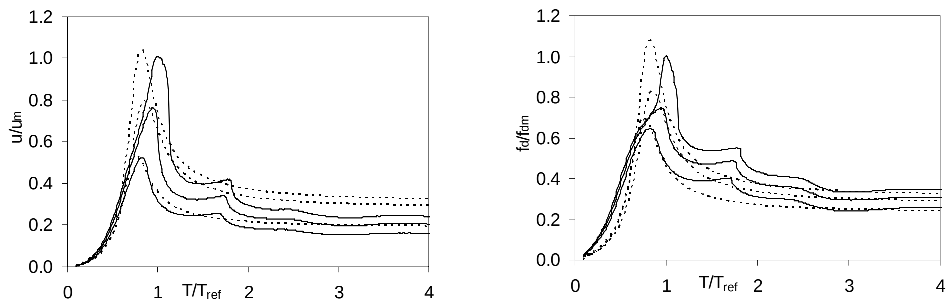

Figure 1 Harmonic analysis: non linear (solid line) and linear (dashed line) systems – case b

The equivalent linear system is obtained by adopting the following rules:

i. The periodic motions of the two systems at the frequencies at which the maximum displacements are attained are selected as similar dynamic situations to apply the equivalence conditions

ii. The stiffness of the linear system is obtained as the secant stiffness observed in the nonlinear motion at the instant in which the displacement attains the maximum value:

$$k_s^L = \frac{f_d\left(T_m\right)}{u\left(T_m\right)} \tag{10}$$

where T_m is the period of the external force producing the maximum response in the nonlinear system. An equivalent linear stiffness G_s^L may be deduced for the rubber as follows

$$G_s^L = \frac{h}{A} k_s^L \tag{11}$$

iii. The damping coefficient is obtained by requiring that the energies dissipated by the two systems in the two reference motions are the same:

$$W_d^L (T_m^L) = W_d (T_m) \tag{12}$$

where T_m^L is the external force period producing the maximum displacement in the linear system. The damping coefficient has the expression

$$\xi_s^L = \frac{W_d \left(T_m\right)}{2\pi k_s^L {u_s^L}^2} \tag{13}$$

In general, the curves obtained for the approximated linear systems at different levels of external forces adequately describe the maximum values of displacements and forces (Fig. 1). In the case of response curves involving small strains (γ =1.0), the periods at which the response is extreme almost coincide in linear and nonlinear systems. Large differences may be observed for higher strain levels (γ =1.5 e γ =2 .0). Far from the response peak the dynamic response produced by the linear system is quite different from the nonlinear response: the displacements are usually overestimated and the force underestimated. The differences are larger for case *c* (the most deformable system) and smaller for case *a*, (the stiffest system). The secondary peak can obviously not be described by the linear models.

The values of the equivalent stiffness and the equivalent damping ratio are reported in Table 1 for the three cases analyzed and for different levels of strain. From the numerical results it may be observed that the equivalent stiffness significantly decreases by passing from case *a*, which vibrates rapidly and has a larger viscous response, to case *c*, which vibrates slowly. The equivalent stiffness is also influenced by the strain amplitude and decreases when the strain increases.

Contrary to the equivalent stiffness, the equivalent damping ratio does not show a remarkable variation and the values observed are bounded by 0.147 and 0.183. This means that the dissipated energy approximately varies proportionally to the stiffness.

Table 1 Equivalent linear parameters – stable response

	γ=2		γ=1.5		γ=1	
	G_s (*N/mm²*)	ξ_s	G_s (*N/mm²*)	ξ_s	G_s (*N/mm²*)	ξ_s
Case *a*	0.866	0.15	0.857	0.17	1.010	0.17
Case *b*	0.753	0.15	0.746	0.18	0.939	0.18
Case *c*	0.672	0.15	0.660	0.18	0.883	0.18

4. TRANSIENT RESPONSE

The Mullins effect strongly influences the system behaviour in the initial path of the response under external force. In order to analyze this situation the system was subjected to initial conditions consisting of an initial velocity v_0, to which an initial value of kinetic energy $(W_0 = \frac{1}{2} m v_0^2)$ is associated, while the other state variables were taken as being equal to zero.

The three previously defined cases were considered and the initial velocity was calibrated in order to attain the same maximum displacements adopted in the previous section and corresponding to the limit shear strains (γ=2.0, 1.5, 1.0). Figure 2 reports the results of case *b* and describes the time histories of displacement and the device force per unit mass observed in the nonlinear system (solid line) together with the results of the linear approximation (dashed line) . Here again the results are presented in a non-dimensional format. The same values of the reference displacement, device force and time interval were adopted in order to simplify the comparison with the results.

Comparing, the cyclic response and the free vibration motion corresponding to the same limit strain, it is evident that the free system vibrates more rapidly at the initial stage and the maximum value of the device force is notably higher. In other words the system is remarkably stiffer. As the initial velocity reduces, the system becomes stiffer and stiffer.

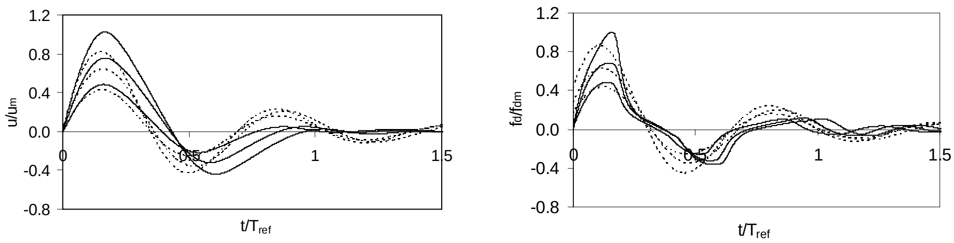

Figure 2 Transient response: non linear (solid line) and linear (dashed line) systems – case *b*

The equivalent linear system is obtained by adopting rules similar to those used in the previous section:

i. The first part of the motions of the two systems up to maximum displacement are selected as similar dynamic situations for application of the equivalence conditions

ii. The stiffness of the linear system is obtained as the secant stiffness observed in the nonlinear motion at the instant in which the displacement attains the maximum value:

$$k_s^L = \frac{f_d\left(t_1\right)}{u\left(t_1\right)} \tag{14}$$

where t_1 is the time at which the maximum displacement of the nonlinear system is reached. An equivalent linear stiffness G_s^L may be deduced for the rubber as follows

$$G_t^L = \frac{h}{A} k_t^L \tag{15}$$

iii. The damping coefficient is obtained by requiring that the energies dissipated by the two systems in the two reference motions are the same:

$$W_d^L(t_1^L) = W_d\,(t_1) \tag{16}$$

where t_1^L is the time at which the maximum displacement of the linear system is reached.

The energy dissipated at the time t_1^L by the linear system is

$$W_d^L(t_1^L) = W_0 - W_e^L(t_1^L) \tag{17}$$

where

$$W_e^L(t_1^L) = \frac{1}{2}\omega^2 u^2\left(t_1^L\right) \tag{18}$$

Consequently from equation (17) it is possible to find the coefficient ξ_t^L once the displacement expression of a linear system subjected to an initial velocity v_0 is introduced in the equation. The displacement expression is

$$u(t) = \frac{v_0}{\omega_s} e^{-\xi\omega t} \sin \omega_s t \tag{19}$$

where $\omega_s = \omega\sqrt{\left(1-\xi^2\right)}$.

The curves obtained by using the linear approximated systems adequately describe maximum values of displacements and forces (Fig. 2) in the case of response curves involving small strains (γ=1.0), whereas large differences may be observed for higher strain levels (γ =1.5 e γ =2 .0). Generally the linear systems are stiffer with respect to non linear systems and are not able to describe the responses after the first quarter of a cycle.

The values of the equivalent stiffness and the equivalent damping ratio are reported in Table 2 for the three cases analyzed and for different levels of strain. From the numerical results it may be observed that the parameters obtained are significantly different from those obtained by considering the stable response. In particular, the equivalent stiffness are much larger especially in case *a* with large strain values (when the Mullin effect is maximum).

Differently the equivalent damping ratio remarkably decreases by passing from the case c to the case *a* and by decreasing the strain.

Table 2 Equivalent linear parameters – transient response

	γ=2		γ=1.5		γ=1	
	G_t (*N/mm*^{2})	ξ_t	G_t (*N/mm*^{2})	ξ_t	G_t (*N/mm*^{2})	ξ_t
Caso *a*	1.202	0.16	1.119	0.14	1.170	0.14
Caso *b*	1.087	0.20	1.005	0.17	1.074	0.16
Caso *c*	0.972	0.18	0.895	0.17	0.972	0.15

5. SEISMIC RESPONSE

In this section the response of the previous three cases subjected to external inputs describing seismic events are analyzed, within the previously considered range of material strains and strain rates. More specifically, seven artificial ground motions are considered. The accelerograms are generated to match the elastic response spectrum given by the OPCM3431 code for zone 1 and ground types B-C-E, according with the rules furnished by the same code.
In order to obtain results which are comparable with others, situations with the same average value of maximum strain equal to γd = 1.5 were considered. In this regard it should be noted that the dynamic properties of the three systems analyzed do not change by modifying area *A* and thickness *h* of the device and mass *m* of the system, if the ratio *A*/*hm* is constant. Consequently, in the three cases analyzed, area *A** and thickness *h** were assigned so as to obtain the desired maximum strain γ_d = 1.5 under the input intensity considered and by maintaining their ratios constant. Table 3 reports the values of the area and the thickness obtained in the three cases and the values of the equivalent stiffness and the equivalent damping coefficient of the respective linear models. In particular the parameters of the linear model related to the stable response (k_s, ξ_s), the parameters of the linear model related to the transient response (k_t, ξ_t) and the average values of previous parameters (k_m, ξ_m) are reported.

Table 3 Geometric characteristics of HDR devices and equivalent linear parameters

	HDR		Lin s		Lin t		Lin m	
	*h** (*mm*)	*A** (*mm*^{2})	k_s (*N/mm*)	ξ_s	k_t (*N/mm*)	ξ_t	k_m (*N/mm*)	ξ_m
Case *a*	14	437920	26800	0.17	35000	0.14	30900	0.16
Case *b*	40	312800	5830	0.18	7860	0.17	6850	0.18
Case *c*	85	166180	1290	0.18	1750	0.17	1520	0.18

The extreme values of displacements and forces obtained by using the non linear model and the three linear models were compared. A useful representation of the results of the different analyses carried out is reported by Fig.3 (case *b*). For each accelerogram the maximum values of displacements and forces obtained by the non linear analysis and the three linear analyses are reported in the abscissa and ordinate axes respectively. In this way the dotted line, which is the bisecting line, indicates when the displacements or forces obtained by using the linear models coincide with those obtained from the non linear analysis.
It may be observed that the linear model referring to the stable behaviour (Lin 1) shows a tendency to overestimate the displacements and underestimate the forces, whereas the linear model referring to the transient behaviour (Lin 2) has the opposite tendency. This is confirmed by the average values reported by Table 4. The results obtained show that the two equivalent models, defined according the criterion proposed, can furnish the extreme values of forces and displacements and do not differ by more than 10% with respect to those obtained by the non linear analysis.

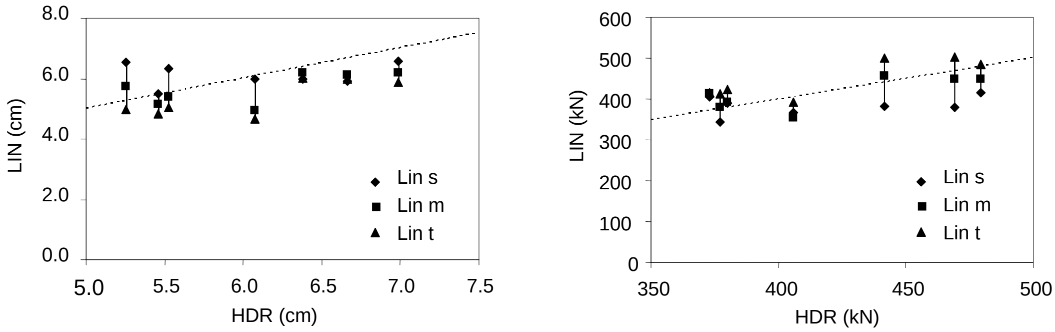

Figure 3 Maximum displacements forces for the seven ground motions considered – case *b*

Table 3 Average results obtained with non linear and linear systems

	Non Linear		Lin s		Lin t		Lin m	
	displ. (cm)	force (kN)	displ. (cm)	force (kN)	displ. (cm)	force (kN)	displ. (cm)	force (kN)
Case *a*	2.14	652	2.27	641	1.96	695	2.08	668
Case *b*	6.05	418	6.12	383	5.34	446	5.67	413
Case *c*	12.75	207	12.49	182	11.31	215	11.95	199

CONCLUSIONS

Linear equivalent models are suggested by current technical codes to model HDR behaviour. These models are generally defined by considering stable behaviour and neglecting the transient response of the material, which strongly influences the seismic behaviour of structures with HDR-based devices, since the extreme values of displacements and velocities are reached only a few times during a seismic event.

This paper proposes a method to define two equivalent linear models, based on stable and transient behaviours. The effectiveness of the proposed models to predict the seismic response was checked by carrying out linear and non linear analyses of simple one-degree-of-freedom systems, in the strain amplitude and strain rate fields that are of interest in seismic design.

The results obtained show that the two equivalent models, defined according the criterion proposed, furnish the extreme force and displacement values with acceptable levels of approximation.

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