

Nonlinear behavior of dynamic systems with high damping rubber devices

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ABSTRACT

The High Damping Rubber (HDR) is widely used in seismic engineering and, more generally, in the passive control of vibrations. Its constitutive behaviour is quite complex and is not simply non-linear with respect to strain but also shows a transient response during which material properties change (Mullins effect). A number of recent works were dedicated to analyzing and modelling the material behaviour. The present work intends to study the consequences of such non-linear behaviour in the dynamic response of S-DoF systems where the restoring force is provided by dissipative devices based on the HDR (structural system with dissipative bracings and isolated systems). Preliminary analyses under harmonic forces and impulsive excitations were carried out in order to separately characterize stable and transient responses. Finally, the response under seismic inputs with different intensities was studied. Results show that the Mullins effect may play an important role in the seismic response and the dynamic properties of the system change significantly for seismic events with different intensities.

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1. Introduction

In the field of seismic engineering the rubber with enhanced dissipating properties, usually known as High Damping Rubber (HDR), is extensively adopted in bearings for the seismic isolation of bridges or buildings [1], and is also used for dissipating devices in order to increase stiffness and the energy dissipation capacity of structures [2–5]. With respect to other types of damper devices, based on elasto-plastic or viscous materials, the HDR-based damper seems to be a promising energy dissipating device because no permanent strains occur after seismic events and moreover it permits dissipating energy even for small lateral displacements produced by wind or minor earthquakes.

Some difficulties in the use of this kind of dissipating device derive from its complex dynamic behaviour which, makes it difficult to evaluate the behaviour of equipped structures accurately and to give design indications. More specifically, the material behaviour is strongly non-linear and both stiffness and damping properties vary with the amplitude of strain and depend on the strain rate [6,7]. Furthermore, the presence of filler added to the natural rubber, makes the response of the HDR strain history-dependent and causes a transient behaviour in which stiffness and damping change remarkably. The phenomenon, usually known as the “Mullins effect” or “scragging”, is a consequence of the damage of the microstructure, that occurs during the process [8,9]. Recent

studies [10] showed that the transient response is related to the maximum strain attained by the material and is influenced by the strain rate. The initial properties of the material may be however recovered (healing effect) and the healing times depend on the material considered and on the temperature [10]. The rubber studied in [11] showed a rapid recovery of a large part of the material stiffness even if the complete recovery may take several months. Consequently seismic analyses of structures endowed with HDR devices should be performed with virgin material properties, even if some scragging process were applied to the devices, and a model with damage should be adopted. The influence of scragging on the seismic response is also evidenced in [12] where the bidimensional response of an isolated bridge is analyzed.

The present work intends to analyze the dynamic response of single degree of freedom systems in which the restoring force is given by HDR devices, by using a unidimensional model based on virgin properties of the rubber and including the scragging phenomenon, previously developed by the authors [11]. The aim of these numerical analyses is to evidence characteristic aspects which can be of interest in the structural design under seismic actions and which cannot be described by simpler models, such as linear visco-elastic or elasto-plastic, usually used to simulate the HDR-based devices behaviour [13,14]. The analyses consider a range of the shear strain from 0.0 to 2.0, which seismic dissipating devices usually undergo. Three different ratios between mass and stiffness have been considered in order to study the rubber response in different dynamic situations, spanning from vibrations with long period, which furnish information about isolated systems, to vibrations with short period, which furnish

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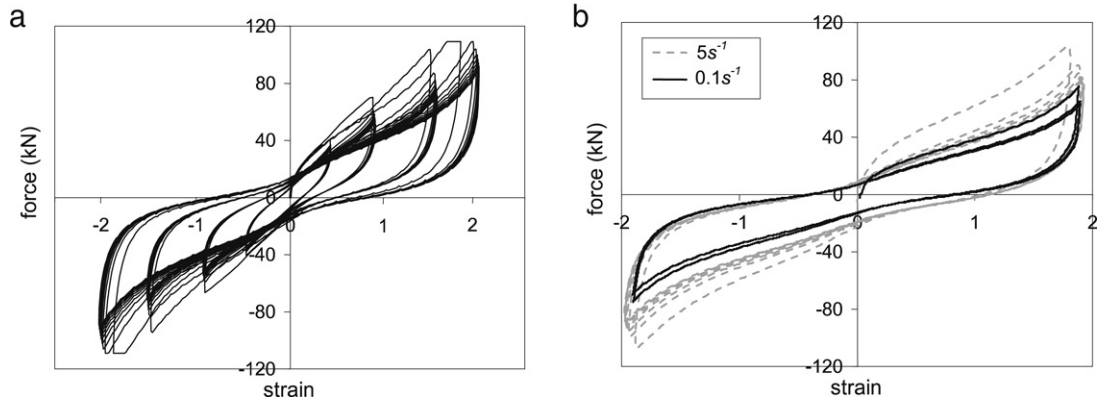


Fig. 1. Mullins effect: different strain amplitudes (a) different strain rate (b).

information about the rubber response in dissipating braces inserted within deformable frames.

In particular, Section 3 investigates the harmonic behaviour of the dynamical systems subjected to sinusoidal forces. The results refer to the stable, post-transient, response and furnish information regarding the influence of non-linear behaviour of the HDR on the dynamic response of the system, once the Mullins effect is over.

The following section is otherwise oriented to highlight the influence of the Mullins effect that influences the initial part of the dynamic response. For this purpose, the system behaviour under an impulsive initial input is studied.

Finally, the last section is dedicated to the analysis of the system subjected to seismic inputs. The analyses show that the Mullins effect may influence and change the system response in the case of similar seismic events (similar frequency content and maximum displacement attained). Furthermore, the influence of all the non-linear phenomena on the response under seismic events of different intensities is analyzed in order to show how the dynamic properties of the system change by varying the input intensity.

2. Dynamical system

2.1. HDR model

The model used to simulate the HDR response is a rheological unidimensional model able to describe the transient response of the rubber, depending on both the strain rate and the maximum strain attained, as evidenced by experimental tests reported by Fig. 1. In the model the *state* of the material is furnished by the shear strain γ , defined as the ratio between the shear displacement and the thickness of the rubber, and by a set of internal variables α_i describing the inelastic response and the Mullins effect. The tangential stress may be derived from the free energy per unit volume $\varphi_d(\gamma; \alpha_i)$ by the relation

$$\tau_d = \frac{\partial \varphi_d}{\partial \gamma} \quad (1)$$

whereas the dissipated power per unit volume w_d may be obtained from the derivative with respect to the internal variables (repeated indexes denote summation, superposed dot denotes time derivative)

$$w_d = \frac{\partial \varphi_d}{\partial \alpha_i} \dot{\alpha}_i. \quad (2)$$

The stress deriving from a strain history may be determined once the initial state and the *process* $\eta = \dot{\gamma}$ are known, on the

basis of the nonlinear functions $g_i(\gamma, \eta; \alpha_i)$, which describe the evolution of the internal variables:

$$\begin{bmatrix} \dot{\gamma} \\ \dot{\alpha}_i \end{bmatrix} = \begin{bmatrix} \eta \\ g_i(\gamma, \eta; \alpha_i) \end{bmatrix}. \quad (3)$$

The expressions of the free energy and those of the evolution functions adopted in the following analyses are reported and commented in the [Appendix](#).

2.2. S-DoF system

The S-DoF (Single-Degree of Freedom) dynamical system considered consists of a mass m and an HDR-based dissipating device that furnishes the restoring force. It is assumed that a linear relation, defined by a constant c , exists between the mass displacement u and the shear strain of the device rubber $\gamma = cu/h$, where h is the thickness of the rubber layer. The value of c depends on the geometry of the connection between the device and mass. The restoring force per unit mass f_d can be expressed in the form

$$f_d = \frac{cA}{m} \tau_d \quad (4)$$

where A is the area of the HDR layer in the device. The *state* of the system is consequently described by the vector $\mathbf{x} = [u, v; \alpha_i]$ where v is the velocity of the mass. The evolution law has the following form:

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{\alpha}_i \end{bmatrix} = \begin{bmatrix} -f_d \left(c \frac{u}{h}, c \frac{v}{h}; \alpha_i \right) + f_e \\ g_i \left(c \frac{u}{h}, c \frac{v}{h}; \alpha_i \right) \end{bmatrix} = \mathbf{A}(\mathbf{x}) \quad (5)$$

where f_e is the external force per unit mass. It may be useful to observe that the following non-linear results may also be extended to cases different from those considered here. The equation of motion can be rewritten by remembering that $u = \gamma h/c$ and by dividing each term by the thickness h . The equation assumes the form

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{\gamma} \\ \dot{v} \\ \dot{\alpha}_i \end{bmatrix} = \begin{bmatrix} \gamma \\ -\frac{cA}{mh} \tau_d(\gamma, \eta; \alpha_i) + \frac{f_e}{h} \\ g_i(\gamma, \eta; \alpha_i) \end{bmatrix} \quad (6)$$

consequently the same strain history may be observed in all those cases where the two ratios cA/mh and f_e/h did not vary (e.g. non-linear response does not vary by doubling f_e if both A and h are also doubled).

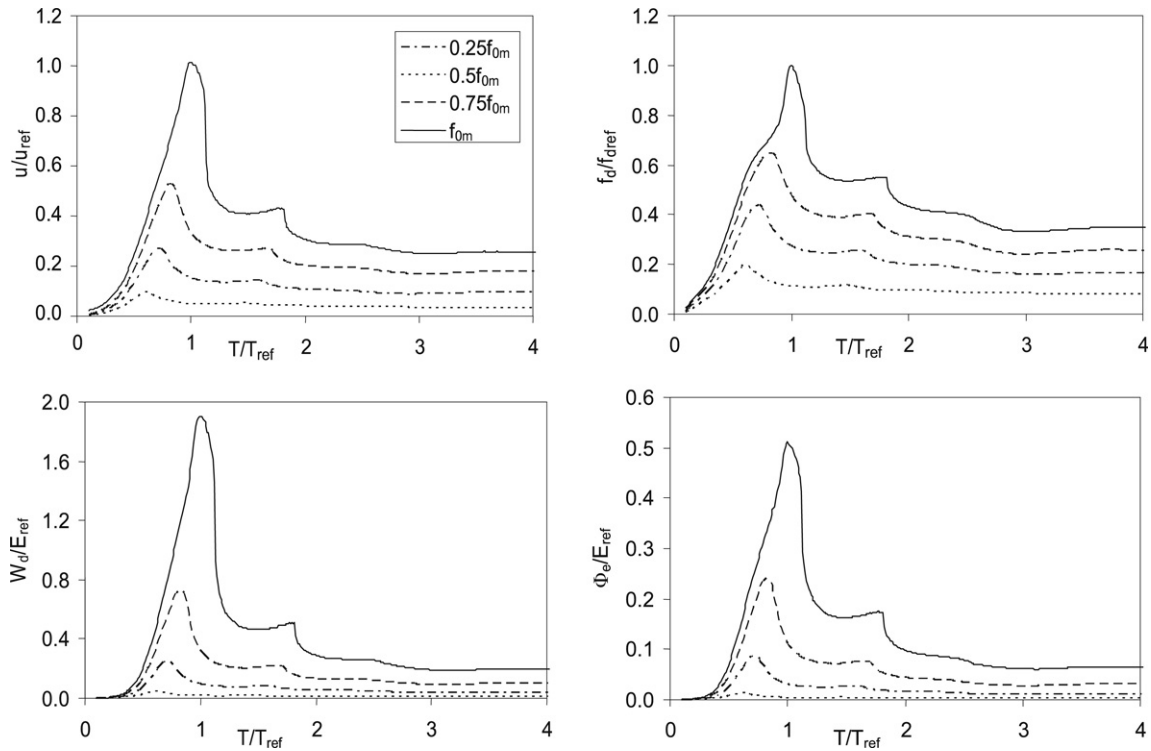


Fig. 2. Harmonic response of case *b*: maximum displacements, maximum forces, dissipated energy and maximum free energy.

3. Harmonic analysis

In this section, a harmonic analysis is carried out in order to investigate the nonlinear response of the system subjected to periodic external force with varying frequency and amplitude.

In the tested range of external actions the system shows a transient response and attains a stable behaviour after a certain number of cycles. The response related to an external action with period T is considered to be stable at the instant t when the difference between the state history observed in the last period and the state history observed in the previous period is sufficiently small. More precisely, once the two state functions $\mathbf{x}_1(t) = \mathbf{x}(t - T + s)$ and $\mathbf{x}_2(t) = \mathbf{x}(t - 2T + s)$ have been defined on the same interval $s \in [0, T]$ and a positive real constant ε has been chosen, the response is considered to be stable if $\|\mathbf{x}_1 - \mathbf{x}_2\| < \varepsilon$. In the application it was assumed that $\|y(s)\| = \max |y(s)|$, $s \in [0, T]$.

Having the aim to characterize the response under seismic events acting on the system where the state variables are null initially, the analyses were performed by assuming $\mathbf{x} = \mathbf{0}$ for $t = 0$. The external force has the expression

$$f_e(t) = f_0 \sin(2\pi t/T) \quad (7)$$

where f_0 is the amplitude of the force per unit mass and T is its period. In particular three cases corresponding to three values of stiffness are considered in the analyses. The intermediate case *b* was obtained by a rubber layer with an area $A = 78\,200 \text{ mm}^2$ and a thickness $h = 10 \text{ mm}$; a mass of 10^5 kg was considered. The other cases *a* and *c* were obtained with a device area four times larger and smaller, respectively. Such values were chosen in order to obtain dynamical systems that show the displacement peak for external force periods of about 0.5 s (case *a*), 1.0 s (case *b*) and 2.0 s (case *c*). The chosen stiffness values make it possible to study different situations referable to different structural systems where devices are introduced in order to reduce seismic effects, like frames with dissipating bracings ($T = 0.5\text{--}1.0 \text{ s}$) or isolated structures ($T = 1.0\text{--}2.0 \text{ s}$). The external force periods considered

in the analyses span from 0.3 to 4.0 s, which is the range of interest in seismic response. Finally, the constant $c = 1$ was assumed in the analyses.

For each case, the maximum intensity of the external force f_{0m} was calibrated to provide a maximum value of shear strain ($\gamma = u/h$) equal to 2.0, which is an usual maximum strain value in the design since larger strains cause a strongly hardening behaviour of the rubber. Other results were also evaluated for smaller values of the maximum strain by considering different intensities of the external force, equal to $0.75f_{0m}$, $0.5f_{0m}$ and $0.25f_{0m}$. The constant c of Eq. (4) was taken as being equal to 1.0.

Fig. 2 reports the maximum values of displacement (u), the restoring forces (f_d) the energy W_d dissipated in a cycle and the extreme values Φ_e attained by the free energy in the periodic response. The diagrams are in a non-dimensional format and were obtained by dividing displacements, forces, energies and periods by reference values defined with the criteria indicated below. The reference displacement u_{ref} and the reference force f_{dref} are the value of the maximum displacement and the value of the maximum device force attained with the external force f_{0m} . The reference period T_{ref} is the period at which these maximum values are reached. The reference energy value is given by

$$E_{ref} = \frac{1}{2} f_{dref} u_{ref}. \quad (8)$$

The results refer to case *b*, for which $f_{0m} = 37 \text{ kN}$, $u_{ref} = 20 \text{ mm}$, $f_{dref} = 1.20 \text{ N kg}^{-1}$ and $T_{ref} = 1.0 \text{ s}$. By observing Fig. 2 it is evident that the main displacement peak occurs at a period (T_m) which decreases by decreasing the force intensity. This is a typical behaviour of softening systems even if the behaviour seems to become weakly hardening for the largest value of the input force, as is usual in the large strain range. The amplitude of the main peak increases nonlinearly with the intensity of the input force, as may be seen in the same figure. This trend is mainly controlled by the dissipation properties of the system, which worsen for large strains.

Fig. 3. Displacement time histories and force–displacement diagrams at the two peaks exhibited by the system: main peak (a), secondary peak (b).

The maximum displacement rapidly changes for periods which are slightly larger than T_m , although multiple responses were not observed in the cases analyzed. Predictably, unstable behaviour may be exhibited by the system for larger strains. It can be also observed that for every value of the input force the response curves have a secondary peak, whose period is about 1.8 times that of the main peak.

In order to analyze the transient response and the shape of the stable loop of the system subjected to harmonic loads a number of analyses in the time domain were reported. Specifically two periods were considered: $T_1 = T_m$ and $T_2 = 1.8T_m$ with the input force f_{0m} . The analysis results, displacement vs time and force vs displacement, are reported by Fig. 3. In the case of $T_1 = T_m$ the displacements are approximately sinusoidal and are strongly amplified. In the case of $T_2 = 1.8T_m$ the displacement history becomes periodic but is no longer sinusoidal. In this case displacement contributions with higher frequencies arise.

Similar trends were observed in the other cases *a* and *c*, even if a number of changes occur. A comparison may be obtained from Fig. 4 where the maximum values of displacements and shear stresses ($\tau_d = f_d/A$) of the previous case *b* are reported together with results of case *a*, which is stiffer and attains maximum values for about $T_m = 0.5$ s, and case *c*, which has a lower stiffness and attains maximum values for about $T_m = 2$ s. The main differences regard the secondary peak and stress intensity, which increase for stiffer system as a consequence of the increment in the strain rate.

In order to furnish synthetic information and to permit a numerical comparison between different situations, three parameters were chosen to describe the system response. The first parameter

$$G = \frac{f_{dm}}{u_m A} \quad (9)$$

is the ratio between the device force and displacement at the instant when the system attains the maximum displacement in the periodic response. It may be interpreted as an approximated estimation of the material stiffness (shear modulus) at the maximum displacement and is strongly dependent on the period T_m .

The other two parameters are derived from energy quantities and describe the dissipative properties of the system. The first parameter is the damping coefficient, which may be defined, in analogy of linear systems, as

$$\xi = \frac{W_{dm}}{4\pi \Phi_{em}} \quad (10)$$

where W_{dm} is the dissipated energy and Φ_{em} is the peak of the energy stored in the system during a period. The previous parameter (ξ) furnishes information on the ratio between dissipated and stored energy and will be evaluated, as usual, for the period at which the system response is at its maximum value, changing case by case.

Sometimes, a different parameter ξ_e (equivalent damping coefficient) in which Φ_{em} is substituted by E_{ref} , was adopted to study rubber behaviour [13–15]. This latter was not used because it has no physical meaning, since E_{ref} does not measure the internal energy of the material.

In order to describe the dissipation rate and, more generally, the global ability of the system to reduce the effects induced by an external input, another parameter

$$\lambda = \frac{2\pi \xi}{T_m} \quad (11)$$

is introduced. This was obtained by dividing the previous parameter by the duration of the time interval on which the dissipated energy is evaluated (average dissipated power). The constant term 2π was added in order to make it similar to the coefficient describing the exponential decay in linear systems. Table 1 reports the results evaluated at T_m and for the cases and the external force levels considered previously.

Predictably, by comparing the results referring to the same case (*a*, *b* or *c*), it may be observed that the shear modulus of the material strongly increases when the maximum strain decreases. On the contrary, the damping coefficient does not change significantly by reducing the external force (the dissipated energy increases in the same way as the free energy) whereas the damping rate coefficient augments as the strain decreases (the energy is dissipated more rapidly). On the other hand by comparing results

