



**RETE DEI LABORATORI UNIVERSITARI
DI INGEGNERIA SISMICA**

RELUIS
Report Scientifico finale

Project of Research N. 6

**INNOVATIVE METHODS FOR THE
DESIGN OF GEOTECHNICAL SYSTEMS**

Coordinated by: Associazione Geotecnica Italiana

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Scientific Report – FINAL REPORT

1. INTRODUCTION

The performance of flexible retaining structures and tunnels subjected to seismic actions can be evaluated with several methods at increasing levels of complexity from pseudo-static methods, or simplified dynamic methods, to fully coupled effective stress numerical analyses under dynamic loading.

In the pseudo-static approach, the earth mass and the structure, be it a flexible retaining wall or a tunnel, are assumed to be subjected to an acceleration which is taken to be constant in space and time and is expressed through a seismic coefficient, K , generally a fraction of the peak acceleration expected at the site. The main conceptual drawback of the method is the selection of a representative value of the seismic coefficient such that the pseudo-static actions produce equivalent effects on the structure as those induced by the seismic actions, i.e. actions that are variable in time and space and transient in nature. In principle an appropriate value of the seismic coefficient should depend on the design ground motion or design ground motion parameters, such as peak ground acceleration, frequency content, and duration, but also on the soil and structure characteristics.

For flexible earth retaining structures, simplified dynamic methods include both displacement-based methods derived from the formulation originally proposed by Newmark for slopes, and subgrade reaction methods. In the latter class of methods, soil-structure interaction is tackled using a decoupled approach in which the ground response is evaluated first in free field conditions and then computed displacements are applied to a set of springs and dashpots restraining the structure. These methods are frequently adopted also for the seismic analysis of tunnel linings, in which the load increments to be applied to the tunnel lining follow from a free-field seismic response analysis. The main problem with the adoption of displacement-based methods for flexible retaining structures is the definition of compatible failure mechanisms, while in subgrade reaction methods there is a problem with the selection of appropriate material parameters for springs and dashpots. Moreover, both for displacement based methods and subgrade reaction methods, representative design input acceleration time histories must be selected.

In principle, numerical methods allow the most comprehensive analysis of the response of flexible retaining structures and tunnels to seismic loading. However, just like any other numerical analysis, reliable dynamic soil-structure interaction analyses require a number of simplifications and approximations of the problem under examination, including the definition of a representative soil profile, the selection of representative mechanical properties for each layer, assumptions on ground water conditions and initial state of stress, structural geometry and boundary conditions, and the selection of representative design input acceleration time histories. Among all the idealisations, a major role is played by the constitutive model adopted for the soil, which should be able to reproduce the main features of its mechanical behaviour under cyclic loading, such as irreversibility of deformations and incremental non-linearity, hysteretic dissipation of energy, and memory of previous stress history. This can only be achieved adopting advanced constitutive models developed within the framework of bounding surface plasticity, kinematic hardening plasticity and hypoplasticity, generally not included in the libraries of commercial codes and requiring input parameters not routinely measured in field or laboratory tests. In the context of this research project, both simple and advanced numerical analyses are carried out finalised both to a better understanding of soil-structure behaviour under seismic loading and to serve as a reference to develop reliable simplified procedures for design to adopt in practice.

2. ACTIVITIES

The research consists of different activities in the following areas:

- 1 Survey of the existing technical literature on the seismic behaviour of flexible retaining structures and tunnel linings.
- 2 Physical modelling: Centrifuge tests on reduced scale models in order to study the behaviour of shallow tunnels and walls under seismic actions. The tests will be performed in the geotechnical centrifuge of the Schofield Centre, at the Cambridge University Engineering Department (CUED).
- 3 Advanced numerical modelling: Finite Element formulation of equilibrium and dynamic coupled flow for a two-phase medium. Evaluation of the potential of existing finite element codes for carrying out fully coupled dynamic analyses. Evaluation of changes to existing codes to satisfy the specific needs of the research project. Identification of constitutive models capable of reproducing the main aspects of soil behaviour under cyclic and dynamic actions. Implementation of the selected models in existing Finite Element codes.
- 4 Simplified dynamic and dynamic analyses: the results obtained by means of advanced numerical modelling and centrifuge tests will be compared with those from simplified dynamic and dynamic analyses.
- 5 Pseudo-static analyses: the principal aim of the comparison between the results of pseudo-static approach and the dynamic analyses is the definition of criteria and procedures to select rationally an appropriate value of the seismic coefficient to be used in pseudo-static methods.

3. RESULTS

3.1. Survey of the existing technical literature

3.1.1. Introduction

During the first 6 months of activities the survey of the technical literature on the seismic behaviour of flexible retaining structures and tunnel lining has been performed by all research units. The results of this activity has been collected in the first semester report and was used as the base of the subsequent studies.

3.2. Centrifuge tests

3.2.1. Introduction

Together with the numerical analyses, several centrifuge tests were carried out on reduced scale models in order to define an experimental database for calibrating simplified methods and advanced numerical analyses of the behaviour of shallow tunnels and walls under seismic actions. The tests were performed in the geotechnical centrifuge of the *Schofield Centre*, at the *Cambridge University Engineering Department* (CUED).

According to technical specifications of this centrifuge, the dynamic tests can be carried out with accelerations up to 80 g. The input signals are sinusoidal waves of several frequencies. The same model underwent different signals of various amplitude and frequency.

The total number of tests is 13 (4 on tunnels + 9 on retaining walls).

All structures have been examined in plane strain. The containers where the models have been realised are two Equivalent Shear Beams (ESB) and the Laminar Box (LB), all equipped to minimise the wave reflection at the boundary and reduce friction between the strongbox wall and the model.

Dry sand was used to realise the model.

The seismic inputs are sinusoidal waves of several frequencies. The same model has been tested under several input signals.

Four models have been tested. The sequence of construction phases was not modelled, but the procedures to create the models were specified in order to include them in the numerical models of the experiments. The four tests have been performed on dry sand models at two different values of relative density.

In the table 3.2.2.1 the programme is shown. In the table, D is the diameter of the tunnel, C is the cover, D_r is the relative density and N is the level of g .

Tabella 3.2.1.1: Centrifuge tests on tunnels

model	D (mm)	C (mm)	D_r	N
T1	75	75	~75%	80
T2	75	75	~40%	80
T3	75	150	~75%	80
T4	75	150	~40%	80

The behaviour of model retaining walls was examined in the transverse section, in plane strain conditions. The total length of the retaining walls was kept constant in the tests and equal to 8 m at the prototype scale. Tables I and II summarise the adopted values of depth of excavation, h , embedment, d , depth of deformable layer, z , distance between walls, B , all at the model scale, relative density D_r and ratio between centrifuge acceleration and gravity, N . The experimental programme that was carried out differs slightly from that originally proposed, due to the necessity of repeating two tests on flexible retaining walls to evaluate the effect of experimental procedures on the quality of the results. The programme included a total of nine tests on models of pairs of retaining walls, in dry sand reconstituted at two values of relative density, six of which cantilevered and three of which propped against each other. Construction sequences are not modelled, but the procedures adopted to create the models are specified to make it possible realistic numerical modelling of the tests.

The total length of the retaining walls was kept constant in the tests and equal to 8 m at the prototype scale. Tables 3.2.1.2 and 3.2.1.3 summarise the adopted values of depth of excavation, h , embedment, d , depth of deformable layer, z , distance between walls, B , all at the model scale, relative density D_r and ratio between centrifuge acceleration and gravity, N .

Tabella 3.2.1.2. Cantilevered retaining walls – experimental programme

prova	h [mm]	d [mm]	z [mm]	B [mm]	D_r	N
CW 1	50	50	200	75	84%	80
CW 2	50	50	200	75	53%	80
CW 3	50	50	200	100	73%	80
CW 4	50	50	200	100	55%	80
CW 5	50	50	200	75	49%	80
CW 6	50	50	200	100	69%	80

Tabella 3.2.1.3. Propped retaining walls – experimental programme

prova	h [mm]	d [mm]	z [mm]	B [mm]	D_r	N
PW 1	140	60	400	150	78%	40
PW 2	140	60	400	150	42%	40
PW 4	140	60	400	200	44%	40

Retaining walls are modelled with aluminum alloy plates of a thickness $s = 3.18$ mm for the cantilevered walls and $s = 6.36$ mm for the propped walls, in order to obtain the same bending stiffness EI of reinforced concrete diaphragm wall with $s = 500$ mm. The props are located 5 mm from the top of propped retaining walls at a distance of 195 mm from each other. They are connected to the walls by cylindrical hinges and can rotate in the vertical plane.

The models were instrumented to measure deformation, axial load in the props and bending moments in the walls, acceleration at different locations in the model or on its boundaries, acceleration of the walls, horizontal displacements of the walls.

Models are prepared using a standard fine dry silica sand, namely Leighton Buzzard Sand 100/70, fraction E, reconstituted at two values of relative density, $D_r \square 50\%$ e $D_r \square 80\%$. Specific gravity of the sand is $G_s = 2.65$, while maximum and minimum voids ratio are equal to 1.014 e 0.613 respectively (Tan, 1990; Jeyatharan, 1991).

3.2.2. Tunnels

Four tests were performed on tunnel models in dry sand (Leighton Buzzard Sand fraction E) at two different values of relative density, in plane strain conditions.

The diameter of the tunnel was kept constant, whereas two different relative positions of the rigid base, the tunnel and ground surface were considered for each relative density by varying the tunnel cover (Lanzano & Madabhushi, 2007a , b).

Several earthquakes were excited during each test and the time histories of acceleration were monitored along different arrays of accelerometers.

The seismic actuator enables the application of series of pseudo-harmonic shear loads during the same centrifuge flight, controlling amplitude, frequency and duration of the input signal (Madabhushi et al. 1998).

The models were created in the Laminar Box, a container designed to guarantee as closely as possible an one-dimensional shear wave propagation in the soil layer. More details on the actuator and the box are available in Lanzano, 2009.

Each test model was stepwise spun up to 80g of gravity and underwent four seismic events (EQ1 to EQ4) at increasing frequency and acceleration amplitudes.

The layout and the preparation of the models were described in the 2° year report.

Each model was instrumented with miniature piezoelectric accelerometers, deployed in the soil and along the model container. The accelerometers were located along 3 different alignments: the first at the middle axis of the tunnel, the second between in an intermediate position between the tunnel axis and the lateral boundary (free field alignment) and the third on the lateral boundary (reference alignment). The input signal was measured by means of another accelerometer located at the base of the box.

The surface settlements were measured by LVDTs, placed in two gantries above the model. The tube has been instrumented with strain gauges in order to measure bending moments and hoop stresses at 4 locations along 2 transverse sections. It has been decided that the main instrumented section will be located at the mid-span of the tube and a second section at 50 mm aside. This second section is needed for two

reasons: checking the plane strain behaviour of the tunnel model (that is bending moment and hoop stress at corresponding locations of different sections should be the same) and for redundancy of experimental data. In total 16 Wheatstone bridges have been attached to the tube and wired (2 force measurements in 4 locations of 2 sections).

The results of the tests are reported in the PhD Thesis of Lanzano (2009) and in the annex on "Centrifuge testing" to this report.

The principal results of the experimental programme are:

1. Acceleration profiles show that the maximum amplification was registered at the reference alignment, while along the free-field and the tunnel alignment the amplification was less significant, showing, in this last case, a filter effect of the tunnel.
2. The LVDTs measures at two symmetrical points on the model surface show that densification occurred in the sand model during the tests. A similar behaviour was observed on all the models, the measurements on the loose sand model showing settlement about twice larger than those on dense sand models. After the swing down stages to 1g, the cumulated settlements were only partially recovered.
3. The time histories of bending moment and hoop force measured in the tunnel lining show the accumulation of stresses in the lining during the earthquake, probably caused by soil densification, which gives rise to significant residual values.

3.2.3. Retaining walls

Tests on retaining wall models were carried out in Equivalent Shear Beam (ESB) containers, consisting of a series of overlapping rectangular aluminum-alloy frames connected by rubber layers in order to obtain a stiffness comparable to that of the soil in the container (Teymur & Madabhushi, 2003). The base of the containers is rigidly connected to the shaking table. The internal dimensions of the containers are $670 \times 250 \times 430 \text{ mm}^3$ for the large container and $560 \times 220 \times 240 \text{ mm}^3$ for the small container; their weight is 1035 N and 545 N for the large and the small container, respectively. The large container was used for tests on cantilevered walls ($N = 40$), the small container for tests on propped walls ($N = 80$). The volume of modelled soil at the scale of the prototype is $26.8 \times 10.0 \times 17.2 \text{ m}^3$ and $44.8 \times 17.6 \times 19.2 \text{ m}^3$, for the large and the small container, respectively.

Models are prepared using a standard fine dry silica sand, namely Leighton Buzzard Sand 100/70, fraction E, reconstituted at two values of relative density, $Dr \cong 50\%$ e $Dr \cong 80\%$. Specific gravity of the sand is $G_s = 2.65$, while maximum and minimum voids ratio are equal to 1.014 e 0.613 respectively (Tan, 1990; Jeyatharan, 1991).

Retaining walls are modelled with aluminum alloy plates of a thickness $s = 3.18 \text{ mm}$ for the cantilevered walls and $s = 6.36 \text{ mm}$ for the propped walls, in order to obtain the same bending stiffness EI of reinforced concrete diaphragm wall with $s = 500 \text{ mm}$. The props are located 5 mm from the top of propped retaining walls at a distance of 195 mm from each other. They are connected to the walls by cylindrical hinges and can rotate in the vertical plane.

The models were instrumented to measure deformation, axial load in the props and bending moments in the walls, acceleration at different locations in the model or on its boundaries, acceleration of the walls, horizontal displacements of the walls.

Ground and container accelerations were measured using miniaturised piezoelectric accelerometers, manufactured by D.J. Birchall Ltd., with a weight of 5 g. The resonant frequency of these transducers is 50 Hz and the maximum error is 5% of full range. Horizontal and vertical accelerations of the walls were

measured using MEMS accelerometers, manufactured by Analog Devices. The resonant frequency of these transducers is 24 Hz and the maximum error is 5% of full range. Horizontal acceleration were measured using transducers type ADXL78 MEMS with a full range of $\pm 35g$, while vertical accelerations were measured using transducers type ADXL193 MEMS with a full range of $\pm 120g$.

The bending moment in the walls was measured using 6 strain gauges glued to the middle section of each wall.

Horizontal displacements of the walls were measured using LVDT transducers held in place by a bar located at the top of the container. The axial load in the props was measured by two miniaturized load cells (diameter 12.7 mm) manufactured by Novatech, type F259, with a full scale range of 1kN.

The layout of each of the 9 tested models and the characteristics of the applied earthquakes are detailed in the Data Reports of individual tests (Conti e Madabhushi, 2007a, b, c, d e 2008 e, f, g, h, i) and summarised in the Scientific report of research activities carried out in the 3rd year of Research by U.R. Roma Tor Vergata (Viggiani, 2009).

The main results of the experimental programme are:

1. The amplifications in terms of maximum acceleration are bigger in the vicinity of the retaining walls than in free-field locations.
2. The amplifications in terms of Arias intensity are bigger than the ones in terms of maximum acceleration, and they are bigger in resonant conditions.
3. During the dynamic phases the cantilevered retaining structures experiment increasing permanent displacements toward the excavation. The permanent displacements increase gradually during each earthquake reaching a plateau; between two subsequent earthquakes the permanent displacements increase only if the subsequent earthquake has a maximum acceleration bigger than the previous one. The displacements depends not only on the current earthquake, but also on the history of accelerations suffered by the structure.
4. Also the bending moments depend not only on the current earthquake, but also on the history of accelerations suffered by the structure.

3.3. Advanced numerical modelling

The main goals of the numerical modeling activities have been twofold: i) obtaining a better understanding of the seismic behaviour of the soil-retaining structure system; and, ii) to use the results as a benchmark to develop simplified analysis methods to be used in current seismic design of such geotechnical structures.

In particular, the research has been developed considering the following main stages:

1) FE formulation of the coupled dynamic flow and equilibrium problem for a two-phase material.

This has required:

- a) a review of the different computational strategies which might be used to perform a complete dynamic analysis of a flexible retaining structure under earthquake loading, with special reference to the way in which the dynamic interactions between the solid and fluid phases of the granular media are taken into account;
- b) a critical assessment of the current capabilities of the existing FE codes – which could be used as main computational platforms for performing the advanced numerical simulations – in light of the considerations made in point (1a);
- c) an evaluation of the modifications which would be necessary to introduce into the available FE platforms to meet the requirements of the project, considering both commercial codes – allowing for full or limited

customization of the material and element libraries – and research-oriented open source codes which can be modified at the source code level.

2) Constitutive modelling of soils subject to earthquake loading.

This has required:

- a) a review of the relevant literature on the formulation and numerical implementation of dynamic coupled consolidation problems, for saturated and unsaturated soils;
- b) a review of the available experimental evidence on the cyclic/dynamic response of granular materials, with particular emphasis on fine-grained soils;
- c) a comparative evaluation of a number of different mathematical approaches for the constitutive modelling of fine-grained soils, specifically developed to reproduce the main features of the mechanical response of such materials under the (cyclic/dynamic) loading conditions which typically occur during earthquakes;
- d) the development of suitable integration algorithms for the numerical implementation of some advanced constitutive models for clays into existing commercial or research-oriented FE codes. This activity represents by far the key issue, as far as the customization of available FE platform is concerned.

3) Dynamic FE analyses using conventional approaches and commercial software platforms.

The objective of this part of the research activity – carried out in cooperation with the other OU's of the Line 6.1 – has been to provide useful information concerning the current capabilities of commercial FE platforms, widely used in the professional practice, with particular reference to:

- a) formulation of the dynamic problem (coupled, uncoupled);
- b) constitutive models available for the soil;
- c) constitutive models for the soil-structure interface;
- d) constitutive models available for structural members, anchors, etc.;
- e) specific aspects of the implementation, such as: i) algorithms employed for the integration in time of the discrete equation of motion; ii) availability of special techniques to reproduce silent boundary conditions; iii) availability of damping relations different from the classical Rayleigh approach widely used in computational dynamics.

4) Dynamic FE analyses using advanced soil models.

In this part of the research activity, the algorithms developed during the stage (2) have been used to carry out advanced numerical simulations of the small-scale centrifuge model tests carried out by other OU of the Line 6.1. In particular, the attention has been focused on the models of shallow tunnels driven in loose dry sands.

3.3.1. General features of constitutive modelling for seismic analysis

Any advanced numerical analysis of the soil-structure interaction processes which occur in a flexible retaining structure during earthquake loading requires a proper description of the main features of the mechanical response of the soil under cyclic/dynamic conditions. The available experimental evidence indicates that, among these, such features as:

- i) nonlinearity;
- ii) irreversibility and incremental nonlinearity;
- iii) hysteretic behavior;
- iv) memory of previous loading history,

might have a paramount effect on the computed response. While classical elastoplastic models for pressure-sensitive granular materials are fairly adequate to reproduce the nonlinearity and irreversible behavior as observed in monotonic loading conditions, they fail to correctly describe the accumulation of

irreversible strains and excess pore pressures which are typically associated with cyclic high-frequency loads, and possess only a limited capability of accounting for the effects of previous loading history.

A number of different models have been proposed in the last 20 years to model the cyclic/dynamic behavior of granular materials. Such models have been constructed based on the following alternative approaches: i) the modification of some of the basic assumptions of classical plasticity; ii) the development of incrementally non-linear constitutive equations directly founded on the basic requirements of nonlinear continuum mechanics, which can be considered as extensions of Truesdell's theory of hypoelasticity. Both approaches have been considered for both fine-grained and coarse-grained soils.

3.3.2. Constitutive modeling of fine-grained soils

For the modeling of cyclic/dynamic behaviour of fine-grained soils, the following criteria for the selection of suitable constitutive models have been adopted:

- a) Simplicity of the mathematical structure of the constitutive equations, which is essential in order to: i) perform a qualitative evaluation of the effects played by the different material constants on the predicted response of the material; ii) to identify the appropriate strategies for the experimental determination of such constants; iii) allow a simple numerical implementation in the selected FE platforms.
- b) Possibility of calibrating the model by means of standard laboratory test results, using direct identification procedures as much as possible.
- c) Ability of reproducing the behaviour of NC and OC soils with a single set of material parameters.
- d) Ability of reproducing Critical State conditions for extreme deviatoric straining.
- e) Ability of reproducing, both from a qualitative and quantitative point of view, the decay of shear stiffness and the increase of hysteretic energy dissipation as the level of cyclic deviatoric strain is increased.
- f) Ability of reproducing, both from a qualitative and quantitative point of view, the destructuration effects experimentally observed in bonded ("structured") natural soils (Burland 1990).

Taking into account the above selection criteria, the research has been focused on the following three classes of constitutive models:

- a) the theory of Bounding Surface plasticity with radial mapping (BSRM; see, e.g., Dafalias 1986);
- b) the theory of Bounding Surface plasticity with multiple, kinematic hardening loading surfaces (BSKH; derived from Mroz 1967);
- c) the theory of Hypoplasticity (HP), outlined in Kolymbas (1991).

In particular, reference has been made to the following three specific models, one for each of the above mentioned classes:

1. the BSRM model of Tamagnini & D'Elia (1999);
2. the BSKH model of Rouainia & Wood (2000);
3. the HP model of Masin (2005), equipped with the intergranular strain concept (Niemunis & Herle 1997).

While the first one is a quite simple isotropic-hardening model with diffuse plasticity and hysteresis, the others adopt one tensorial internal variable to reproduce short-range memory effects, and therefore describe an anisotropic behaviour.

Although the mathematical structure of the three models is quite different, they all share the basic principles of Critical State soil mechanics, and are capable to reproduce such features of fine-grained soils

as: i) the existence of a virgin compression line; ii) the existence of a critical state locus which is asymptotically reached for very large deviatoric strains. For this reason, most of the material constants characterizing the three models in monotonic loading conditions share the same physical interpretation, and therefore assume the same value when the models are independently calibrated from the same set of experimental data. This guarantees a minimum necessary level of consistency when the comparative evaluation of the predictive capabilities of each model for a given IBVP is of concern.

Finally, all the three models are capable of describing the mechanical effects of “bonding” as observed in natural soils, and the destructuration effects taking place when the bonding is progressively lost due to mechanical loading.

3.3.3. Constitutive modeling of coarse-grained soils

For the modeling of cyclic/dynamic behaviour of coarse-grained soils, the following criteria for the selection of suitable constitutive models have been adopted:

- a) Simplicity of the mathematical structure of the constitutive equations, which is essential in order to: i) perform a qualitative evaluation of the effects played by the different material constants on the predicted response of the material; ii) to identify the appropriate strategies for the experimental determination of such constants; iii) allow a simple numerical implementation in the selected FE platforms.
- b) Possibility of calibrating the model by means of standard laboratory test results, using direct identification procedures as much as possible.
- c) Ability of reproducing the behaviour of loose and dense soils with a single set of material parameters.
- d) Ability of reproducing Critical State conditions for extreme deviatoric straining.
- e) Ability of reproducing, both from a qualitative and quantitative point of view, the decay of shear stiffness and the increase of hysteretic energy dissipation as the level of cyclic deviatoric strain is increased.
- f) Ability of reproducing, both from a qualitative and quantitative point of view, the phenomena of static and cyclic liquefaction due to the accumulation of excess pore pressures in undrained or partially drained conditions.
- g) Ability of reproducing, at least qualitatively, the effects of induced anisotropy and of the evolution of material fabric during the cyclic deformation process.

Taking into account the above selection criteria, the research has been focused on the following two groups of constitutive models:

1. The BSKH elastoplastic model of Manzari & Dafalias (1997) and its successive modifications of Papadimitriou & Bouckovalas (2002) and Dafalias & Manzari (2004);
2. The HP model of von Wolffersdorff (1997), equipped with the intergranular strain concept (Niemunis & Herle 1997).

Such models represent a reasonable compromise between the contrasting requirements of simplicity and versatility in reproducing the fundamental aspect of the observed cyclic/dynamic response of sands. A further advantage of the BSKH approach selected lies in the possibility of using the three different models as hierarchical enrichments of the version of the basic Manzari & Dafalias (1997) model, and of evaluating the impact of the different options through parametric studies.

3.3.4. Numerical implementation

A common feature of the constitutive models developed in the frameworks of BS plasticity and hypoplasticity is the fact that they are all cast in an incremental form. Rather than providing the state of stress associated to a specific state of strain, they define the evolution laws for the state variables. Therefore, the quantitative evaluation of the mechanical effects of a given “load”, be it an imposed stress increment, strain increment or a combination of both, requires the solution of an initial value problem, consisting in the integration of the constitutive equation along the assigned loading path, with prescribed initial conditions. As this task cannot be performed analytically, except in very special cases, the development of a numerical algorithm is a crucial part of any computational procedure for the solution of nonlinear problems in geomechanics.

More specifically, in the application of the FE method to the solution of a nonlinear IBVP, the following general strategy is usually adopted:

- 1) from the original system of governing PDE's, a nonlinear system of algebraic balance equations is obtained by the introduction of appropriate space and time (if required) discretizations. Such a system is typically solved by adopting an incremental-iterative approach;
- 2) for any given global iteration, the discretized equilibrium equations generate incremental motions, which, in turn, are used to determine the incremental strain history by purely kinematic relationships;
- 3) for a given strain increment, updated values of the state variables are obtained by integrating numerically the constitutive equations at the local level, with given initial conditions;
- 4) the discrete balance equations are then checked for convergence, and if the convergence criterion is not met, the iteration process is continued by returning to step (2).

In particular, the integration of the constitutive equation at the local level – i.e., step (3) – represents the central problem of computational inelasticity, since it corresponds to the main role played by the constitutive equation in actual computations. There are of course many other important computational ingredients in the overall procedure, but they are particular to the type of solution strategy employed, and involve the constitutive theory only in a limited way, if at all. Moreover, the precision with which the constitutive equations are integrated has a direct impact on the overall accuracy of the analysis.

3.4. Advanced numerical analyses

The activities of the third year in the advanced numerical modelling are:

1. Implementation of advanced numerical models in FE codes (ABAQUS, FEAP);
2. Calibration of the Papadimitriou & Bouckovalas (2002) model for the Leighton Buzzard Sand, on the data obtained through experimental tests (TX-CD, TX-CU, TS and RC) by Santucci de Magistris & Visone (2008);
3. Modellazione agli EF di prove su modello in scala ridotta, in condizioni di gravità artificiale, relative allo scavo di gallerie circolari in sabbia con differente densità relative iniziali (Lanzano 2008, Bilotta 2008, Viggiani 2008).

3.4.1. Verification of the implementation of advanced numerical models

The verification of the implementation of advanced constitutive models has been performed in two steps::

1. Comparisons between the results of FEM simulations of typical laboratory tests (oedometer and triaxial tests) and the results of constitutive drivers able to reproduce stress paths deduced from literature (Niemunis, 2008);

2. Use of FEM implementation to reproduce literature results.

3.4.2. Calibration of the Papadimitriou & Bouckovalas model

The calibration of the Papadimitriou & Bouckovalas (2002) model for the Leighton Buzzard sand has been carried out based on the data obtained by Visone (2008) in the experimental program shown in Tab. 3.4.2.1. The Papadimitriou & Bouckovalas (2002) model is characterized by 16 material constants, as shown in Tab. 3.4.2.2. The values of these constants are reported in Tab. 3.4.2.3.

Tabella 3.4.2.1 – Dati sperimentali

Sigla prova	Tipo prova	p'_0 (kPa)	D_r iniziale	e_0
LBS02	TX-CU	200	29%	-
LBS03	TX-CU	400	31%	-
LBS05	TX-EU	200	28%	-
LBS06	TX-EU	400	30%	-
LBS07	TX-CD	100	81%	-
LBS08	TX-CD	200	70%	-
LBS13	TX-CDp	100	76%	-
LBS14	TX-CDp	200	77%	-
LBS15	TX-CDp	400	81%	-
LBSand02	RC	30-400	-	0.7824
LBSand03	RC	30-100	-	0.8250
LBSand04	RC	40-200	-	0.8037
LBSand06	RC	50-400	-	0.7280
LBSand03	TS	100	-	0.8208
LBSand04	TS	200	-	0.7969
LBSand06	TS	400	-	0.7139

Tabella 3.4.2.2 – Costanti del modello

Parametro	Definizione
p_a	Atmospheric pressure
e_{CSa}	Void ratio on CSL at $p=p_a$
λ	Slope of CSL
M_c	Slope of CSL in q:p plane, TX compression
M_e	Slope of CSL in q:p plane, TX extension
m	Opening of yield surface cone
B	Shear modulus constant
ν	Poisson's ratio
a_1	Shear modulus decay parameter
γ_1	Shear strain threshold
k_c^b	Coefficient for BS cone, TX compression
k_c^d	Coefficient for D cone, TX compression
A_0	Dilatancy constant
h_0	Hardening modulus constant

H_{0f}	Fabric index constant
ζ	Fabric index constant

Tabella 3.4.2.3 – Valori delle costanti del modello

Parametro	Definizione	Valore assunto
p_a	Atmospheric pressure	98.1
e_{CSa}	Void ratio on CSL at $p=p_a$	0.822
λ	Slope of CSL	0.0357
M_c	Slope of CSL in q:p plane, TX compression	1.343
M_e	Slope of CSL in q:p plane, TX extension	0.867
m	Opening of yield surface cone	0.0672
B	Shear modulus constant	100.0 ^(*)
ν	Poisson's ratio	0.3
a_1	Shear modulus decay parameter	0.85
γ_1	Shear strain threshold	0.00025
k_c^b	Coefficient for BS cone, TX compression	4.04
k_c^d	Coefficient for D cone, TX compression	4.66
A_0	Dilatancy constant	0.7 ^(*)
h_0	Hardening modulus constant	600.0 ^(*)
H_{of}	Fabric index constant	68000.0 ^(*)
ζ	Fabric index constant	1.0 ^(*)

(*) Valori determinati per tentativi.

3.4.3. Gallerie

The model of Papadimitriou & Bouckovalas (2002) has been used for FEM simulations of the results of centrifuge tests performed at the University of Cambridge. In the FE simulation of the test T1-EQ1, the code ABAQUS Standard v6.7 has been used. The FE mesh adopted in the simulation is shown in Fig. 5.2. It is composed of 352 Q8P0 plane strain elements for the soil and B3 beam elements for the tunnel lining, for a total of 1125 nodes and 2154 dofs.

The FE simulation of the centrifuge test has been carried out in four stages, according to the following scheme:

- 1) Application of gravity (1g) and generation of the initial geostatic state (in terms of stress and internal variables), under quasi-static conditions;
- 2) Removal of the elements inside the excavation perimeter, and simultaneous activation of the liner elements under quasi-static conditions;
- 3) Application of the artificial gravity field (80g), under quasi-static conditions
- 4) Application of the prescribed accelerogram at the model base, under fully dynamic conditions.

The time history of the horizontal accelerations imposed at the model base during stage (4) corresponds to the accelerations registered at the accelerogram located at the model base during the first dynamic stage of the centrifuge test (EQ1).

The results of the simulation indicate that there is a clear amplification effect of the horizontal accelerations, moving from the model base to the ground surface. The amplification effect is significant in a medium-high frequency range, from 60 to 85 Hz. This effect is clearly shown by the Fourier spectra. On the contrary, the computed time histories of the horizontal accelerations do not appear very much affected by the distance x from the tunnel axis.

The evolution with time of the bending moments on the tunnel lining in four points placed along the diagonals at 45° from the horizontal and vertical axes show that the magnitude of the bending moments in all points tends to increase almost monotonically with time. This is due to the progressive accumulation of plastic strains in the soil surrounding the tunnel lining during the seismic stage. Such an effect can be observed on the entire lining. Before the seismic excitation bending moments are quite small; at the end of the seismic stage the increments in bending moments can be as large as 600%, at the tunnel flanks.

3.4.4. Flexible retaining walls

For flexible retaining structures only conventional FEM analyses on simplified models have been performed, since the experimental programme was completed too late to allow the interpretation of the results through advanced numerical modelling.

3.5. Numerical analyses with commercial codes

A more promising strategy to analyze the seismic response of tunnels or flexible retaining structures is to perform a complete soil-structure interaction analysis, based on one of the several FE or FD computer codes specifically developed for geotechnical engineering applications nowadays available on the market at a relatively low price, and thus easily accessible to practitioners.

However, using such tools to perform a complete soil-structure interaction analysis is a very complex task which requires careful consideration of a number of important issues which may have a dramatic impact on the computed results.

From the numerical standpoint, two major issues are the modelling of the radiation condition via proper boundary conditions and the choice of the integration method used to advance the solution in time. From the mechanical standpoint, careful consideration should be given to; i) the appropriate definition of the seismic input; ii) the modelling of soil/structure interface behavior; and, most important, iii) the constitutive modelling of the cyclic/dynamic behavior of the soil.

About this last point, it is worth noting that one of the major drawback of commercially available FE or FD codes is the lack of any constitutive model capable to simulate with sufficient accuracy the cyclic and dynamic response of natural soils in their material library, most likely due to the inherent complexity of such models and the large number of experimental data required for their calibration. sofisticate sono stati confrontati con quelli ricavabili da analisi numeriche complete e semplificate.

3.5.1. Tunnels

As it regards tunnels, the activities of the II years have been focused on complete dynamic analyses of kinematic interaction between a circular tunnel and the surrounding soil, in the transverse and longitudinal section.

Three subsoil profiles were considered, the same already adopted during the first year of the project (*cf* Bilotta *et al.*, 2007a,b,c), which correspond to deposits of typical soft clay, medium-dense sand and gravel. The sample problem was a circular tunnel with reinforced concrete lining, diameter of 6 m and thickness

of 0.3 m, axis at a depth of 15 m in a 30 m thick layer, overlying a relatively stiff bedrock ($V_r = 800$ m/s, $\gamma = 22$ kN/m³, $D_o = 0.5\%$).

The dynamic analyses were performed by means of two computer codes: EERA (Bardet *et al.*, 2000) and Plaxis 8.2 (Brinkgreve, 2002) The code EERA allow to perform free-field seismic response analyses, by modelling the soil as an equivalent linear viscous-elastic medium. Both codes allow the application of input time histories at the bedrock depth. These are taken from a database of Italian seismic events collected by Scasserra *et al.* (2006). The signals have been scaled to values of a_g equal to 0.35g.

The programme EERA was used to calibrate the calculation domain and the parameters to be used in PLAXIS

Standard absorbent elements (Lysmer & Kuhlemeyer 1969) have been placed at the lateral boundaries to simulate the radiation condition.

The distance of lateral boundaries was calibrated to minimize the influence of the boundaries on the obtained results.

In PLAXIS numerical simulations two types of damping exist: numerical damping, due to finite element formulation, and material damping, due to viscous properties, friction and development of plasticity.

In Plaxis, and in the major part of dynamic FE codes the material damping is simulated with the well-known Rayleigh formulation. The damping matrix C is assumed to be proportional to mass matrix M and stiffness matrix K by means of two coefficients α_R and β_R

$$[C] = \alpha_R [M] + \beta_R [K] \quad (1)$$

Different criteria exist to evaluate the Rayleigh coefficients (see for instance Lanzo *et al.*, 2004, Park & Hashash, 2004). In terms of frequency, the dynamic response of a system is affected by the choice of these parameters to a large extent.

In the numerical implementation of dynamic problems, the formulation of time integration constitutes an important factor for stability and accuracy of the calculation process. Explicit and implicit integration are two commonly used time integration schemes. In Plaxis, the Newmark type implicit time integration scheme is implemented. The coefficients α_N and β_N , which have not to be confused with Rayleigh coefficients, determine the accuracy of numerical time integration. For determining these parameters, different suggestions are proposed, too.

In order to perform accurate dynamic analyses in plane strain conditions a correct definition of material (α_R e β_R) and numerical (α_N e β_N) damping is needed.

As a result of numerical simulations, a procedure was set up to calibrate damping parameters in order to reproduce free-field response analysis

3.5.2. Retaining flexible structures

As it regards retaining walls, the activities of the 3 years have been focused on complete dynamic analyses of simple scheme of retaining walls.

Two structural schemes were analysed of one cantilevered retaining wall and one embedded retaining wall with one level of props in dry sand of given mechanical and physical properties, in plane strain conditions. The analyses were carried out using different codes, different constitutive models and, sometimes, different assumptions.

The acceleration time histories have been selected from a database of Italian seismic events collected by Scasserra *et al.* (2006). The signals have been scaled to values of a_g equal to 0.35g.

The research units of Napoli Parthenope and Molise have used the FEm code PLAXIS to analyse cantilevered retaining structures.

The research unit of University of Molise have used a particular procedure to calibrate numerical and material damping, while the research unit of Napoli Parthenope has performed a parametric study to analyze the influence of Raileigh damping on the numerical predictions.

Two different soil models have been adopted for the granular soil: the classical Mohr-Coulomb elastic-perfectly plastic model and the isotropic hardening "Hardening-Soil Model" (HSM), both available in the material models library of PLAXIS®.

The results show that under static conditions, active and passive limit states are fully mobilized down to a depth of about 5 m from the original ground surface. Below this depth, in front of the wall σ_h remains approximately constant, while behind the wall it increases up to the full geostatic value ($K_0 = 0.5$). The seismic shaking produces significant permanent changes in earth pressure distributions on the two sides of the wall. The largest variations occur in the zone located between 4 m (the final excavation level) and 8 m below the original ground surface, where the normal stresses on both sides of the wall are more that doubled.

The results clearly show that, for both values of the damping ratio, the maximum moments and the post-seismic moment are largely greater than the initial, static ones. As expected, the reduction in stress levels experienced by the wall as the damping ratio increases reflects into a decrease of the peak bending moments with D .

The research activities at Roma 'La Sapienza' included a number of numerical analyses using FLAC code to study the seismic behaviour of flexible retaining walls embedded in a coarse-grained soil. Pairs of facing retaining walls were analysed, either cantilevered or connected at the top by a single prop level. Under seismic conditions it is important to avoid connecting the prop level to a fixed point, since no fixed points can be assumed to exist during an earthquake.

The analyses were carried out in plane strain conditions, using the finite difference method. The soil was regarded as an elastic-plastic material with purely hysteretic damping, as discussed in the first-year report; no viscous damping was introduced. Contact between soil and walls was simulated through interfaces with a reduced friction angle. The retaining walls were assumed to be either linearly elastic or elastic-perfectly plastic, to allow for redistribution of bending moments once the yield moment is attained.

Results were interpreted looking at the spatial and temporal distribution of the following quantities: soil-wall contact stresses; acceleration in the investigated domain; permanent displacements; and stresses in walls and props. It was observed that propagation of the two seismic records in the vicinity of the retaining walls causes instantaneous attainments of the available shear strength in soil zones interacting with the walls, resulting in a progressive increase in bending moments, and in a gradual accumulation of displacements.

3.6. Pseudo-static analyses

3.6.1. Tunnels

In the pseudostatic analysis of tunnels four different methods have been developed, to evaluate the maximum shear stress, which can be calculated from the equilibrium of a deformable soil column between the surface and a given depth z . The maximum soil shear strain at a depth z is always calculated by dividing the maximum shear stress, $\tau_{\max}(z)$, by the shear stiffness, $G(z)$, at the same depth:

$$\gamma(z) = \frac{\tau_{\max}(z)}{G(z)} \quad (2)$$

With each pseudo-static method, both linear and linear equivalent analyses have been carried out; in linear analyses the shear modulus G in Eq. (2) coincides with the small strain stiffness G_0 , whereas in linear equivalent analyses it depends on the strain level. For a straightforward evaluation of the maximum shear strain in the soil, the Ramberg & Osgood (1943) model has been used, which expresses the shear strain as

a function of the shear stress as follows:

$$\gamma_{\max}(z) = \frac{\tau_{\max}(z)}{G_0} + C \left[\frac{\tau_{\max}(z)}{G_0} \right]^R \quad (3)$$

In eq.(2), C and R are two parameters that assume the values reported in tab 3.6.1.1, for the three different soils considered in the analyses.

	C	R
Argilla	12000	2.24
Sabbia	800000	2.63
Ghiaia	8000000	2.60

Tabella 3.6.1.1. – Values of the Ramberg-Osgood model.

Method 1

The maximum shear stress to be used in eq. (2) obtained from dynamic equilibrium of a column of soil is:

$$\tau_{\max}(z) = r_d(z) \frac{a_{\max,s}}{g} \sigma_v(z) \quad (4)$$

where σ_v is the total vertical stress; $a_{\max,s}$ is the maximum surface acceleration; r_d is a reduction parameter which takes into account soil deformability. In this work the relationship by Iwasaki (1978) was used:

$$r_d = 1 - 0.015 z \quad (z \text{ in m}) \quad (5)$$

The value of $a_{\max,s}$ is evaluated following the OPCM 3274:

$$a_{\max,s} = S a_g \quad (6)$$

where S is the amplification factor and a_g is the value for outcropping rock.

Method 2

The maximum shear stress is calculated from the equilibrium of a column of soil between the free surface and the depth z, under the hypothesis that the maximum acceleration varies between the value at the bedrock and the value at the soil surface according to an harmonic law.

The acceleration at the bedrock, $a_{\max,b}$, is selected equal to the value for outcropping rock, a_g , deduced from OPCM for the different seismic zones, while the value at the soil surface, $a_{\max,s}$, is evaluated from eq.(6). Between these two values the maximum acceleration varies as:

$$a_{\max}(z) = a_{\max,b} + \text{sen} \left(\frac{2\pi(H-z)}{4H} \right) (a_{\max,s} - a_{\max,b}) \quad (7)$$

The maximum shear stress $\tau_{\max}(z)$ can be deduced from the integral:

$$\tau_{\max}(z) = \int_0^z \rho a_{\max}(z) dz \quad (8)$$

Method 3

The third method assumes a different variation of a_{\max} with depth: namely, the $a(z)$ profile is assumed linear from surface ($S \cdot a_g$) to bedrock (a_g). Such hypothesis corresponds to that of a layer of soil shaking in a range of frequencies much lower than the fundamental.

Method 4

This method is similar to the previous, but the values of r_d have been calculated by using the equation:

$$r_d(z) = \frac{\tau_{\max}(z)}{\sigma_v(z)a_{\max,s} / g} \quad (9)$$

where τ_{\max} was computed by the site seismic response (SSR) analyses, and the value of $a_{\max,s}$ was set again by using Eq. (6); this was necessary to provide results comparable with those obtained by the other methods.

The results of these simplified methods have been compared with those of dynamic analyse with commercial codes. The scope of this study was to set a simplified procedure in the design of tunnels under seismic conditions.

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