Identification and Model Updating in condition of distributed uncertainties

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Unknown input

- It is the case of all the identification techniques based on ambient vibrations. Generally they apply to OPERATIONAL MODAL ANALYSIS.

- Those techniques operate often on low energy signals.

- The response signals are almost, but not necessarily, linear; response parameters are influenced by pre-load conditions.

- Unknown input techniques are important because they allow a cost effective test exploitation and a continuous on-line monitoring action.
Vibration based Identification: what for?

- Numerical model assessment (OMA)
- Prediction of dynamic response (OMA)
- Catching the mechanical parameters evolution and changes (on-line SHM-OMA)
- Experimental knowledge for residual safety evaluation after severe external events like earthquakes (non-linear identification strategies, no OMA)
SHM in CIVIL ENGINEERING:

**Class 1**
- Lightweight modern structures, new high-rise towers, new long-span bridges
  - Flexible, mostly low damped structures;
  - Well characterized and reliable materials;
  - Well known geometry
  - Nearly hyperelastic service behaviour;
  - SHM looking for localized single or multiple defects.

**Class 2**
- Rigid bridges, existing and damaged constructions, ancient heritage
  - Stiff, mostly high-damped structures;
  - Poor, degrade, and non-reliable base materials, with largely scattered mechanical properties;
  - Uncertain geometry;
  - Non-linear behaviour;
  - SHM ?
Class 2, SHM FIRST STEP: EXPLORING THE ACTUAL CONDITIONS:

- local variability of geometric properties and masonry internal organization;
- lack of material continuity, hidden empty volumes
- local variability of the material strength and stiffness, due to original defects or electro-chemical degradation;
- distribution of cracks, subject to thermal path (seasonal width oscillation with basic trend to increase continuously, due to cumulated debris inside the crack);
- effects of past, non documented, damages and repairs, architectural changes, local manipulations.
SECOND STEP: DESIGNING AND ASSESSING THE MONITORING DEVICES AND PROCEDURES

1) Preliminary analyses:
   - risk analysis;
   - optimisation of the measurement network;
   - design of the signal acquisition procedure and sensor choice;
   - dislocation of the permanent sensors in order to obtain the automatic monitoring system be sensitive to possible damage and defects.

2) Realization and assessment:
   - testing of permanent measurement networks on laboratory samples and real existing structural components;
   - positioning supports for temporary or periodic observations.

3) Global on-line testing:
   - dynamic vibrational response measurement acquisition and elaboration;
   - Mechanical identification;
   - symptom based diagnosis;

4) Modelling and Model Updating:
   - FE model based diagnosis;
   - safety evaluations.
   - Model aided identification of faults detectable by monitoring and those that do not show apparent or perceptible forewarnings.
“Robust” monitoring.

“robust” applies to every algorithm, process, method or technique able to reduce the sensitivity of analysis results against input data and measure errors, noise or uncertainties.

In case of large distributed uncertainties “non robust” means often “unable to give a response”
Some basic principles

- Methods able to solve by complex algorithms only simple problems are of none interest
- Reliable methods can treat complex problems with reasonable simplicity (method complexity shall not increase with the complexity of the problem)
- Methods shall survive real experimental validations
- Unfit models cannot be updated
Global (dynamic) testing in detail

- Robust identification algorythm
- Robust model updating technique
Identifying the modal parameters of a linear system

SMART STRUCTURES → ON-LINE MONITORING

OUTPUT ONLY VIBRATION MEASURES
OUTPUT ONLY IDENTIFICATION PROCEDURES
Linear OMA identification

- **Time domain methods** (linear time invariant)
- **Spectral methods**
  - **Frequency domain methods** (linear time invariant)
  - **Time-frequency or time-scale domain methods** (linear time variant)
Modal analysis by time domain techniques

- There are several Time Domain techniques but their conceptual contents are strictly mutually related. The starting point is the Lagrange solution of the dynamic response of linear MIMO mechanical systems.
Response of a MIMO damped linear system

\[ \{\dot{Z}\} = [\Phi]\{Z\} + [\Theta]\{F(t)\} \]

\[ \{Z\} = \left\{ \{\dot{x}\} \right\}; \{F(t)\} = \left\{ \{f(t)\} \right\}; \]

\[ [\Phi] = \left[ \begin{bmatrix} -\{[M]\}^{-1}\{C\} & -\{[M]\}^{-1}\{K\} \end{bmatrix} \right]; \]

\[ [\Theta] = \left[ \begin{bmatrix} \{[M]\}^{-1} & [0] \\ [0] & [0] \end{bmatrix} \right] \]

The general Lagrange solution

\[ \{Z\} = [A]\{Z_0\} + [b] \otimes \{F(t)\} \]

The response \{Z\} in the state space is the sum of two contributions:
1. the free decay after the initial state \{Z_0\}
2. the convolution of the impulse response (free decay) and the external forcing actions.

In the discrete – time domain:

\[ \{Z\}_k = [A]\{Z_0\} + [B]\{F\}; \{Z_0\} \text{ can be related to the any previous time step} \]

\[ \text{e.g.:} \quad \{Z\}_k = [A_k]\{Z\}_{k-1} + [B_k]\{F\}_k \]

In a recursive procedure: \[ \{Z\}_k = [A_k]\{A_{k-1}\}\{Z\}_{k-2} + [B_{k-1}]\{F\}_{k-1} + [B_k]\{F\}_k \ldots \]
The part of the recursive process related to the past responses is called “autoregressive.”

If the input F is not known, it is necessary to assume that its statistic nature is given.

Generally an assumption of stationary white noise input is required.

When each state depends only from the previous-one the process is a recursive Markov process (ERA, Subspace………).

If each state depends from more previous states, the recursive process is non-Markov or semi-Markov; auto-regressive coefficient are overlapped (ARMAV or DSPI approach, much more robust than ERA or Subspace).
Spectral Analysis

Stationary signals:
(FREQUENCY ANALYSIS)

- Fourier Transform,
- Periodogram,
- AR modelling

Non Stationary signals:
(TIME-FREQUENCY ANALYSIS)

- STFT,
- Spectrogram
- Wigner-Ville & T-F distributions of Coehn’s class,
- Evolutionary spectra
- Wavelets

Slow Variations

Rapid Variations
Time-frequency methods

+ high accuracy in parameter estimation
+ possibility of managing effectively non-stationary signals
+ ability in handling moderate non-linearities
+ high robustness against noise (even in high-frequency range)
- heavy computational cost
- higher theoretical and algorithmic complexity

MOREOVER:

Time-Frequency methods supply the spectral content evolution in time. Therefore:

intrinsically fit to continuous on-line monitoring needs
T-f bi-linear transforms: autospectra
Modal responses are not all excited simultaneously! A time window cleverly chosen can help addressing close modes.
It is possible to define the estimator for the phase difference between two signals as:

$$\Delta \varphi_{x,y} (t, f) = \arctg \left\{ \frac{\Im \{ X_{WV}^{x,y}(t, f) \}}{\Re \{ X_{WV}^{x,y}(t, f) \}} \right\}_{f = f_0}$$

consistent for any distribution belonging to the Cohen class.
$\Delta \varphi_{x,y}(t, f)_{f = f_0}$ is expected to be nearly constant versus time if $f_0$ is a modal frequency.
Strategy:

We explore, along the frequency axis, the

\[ St. Dev_t (\Delta \varphi_{x,y}(t, f)_f) , \]

e.g., for each \( f \) value, we compute the standard deviation along the \( t \) axis of the function:

\[ \Delta \varphi_{x,y}(t, f)_f \]

If it approaches zero, then:

\( f \) is a modal frequency
Estimating the modal shape

Given the signals:

\[ s_i, s_j \quad (\text{real recorded contemporary signals}) \]
\[ s_k \quad (\text{virtual pure sinusoidal with } f = f_0) \]

Indicating by:

\[ D_{s_i} (t, f), D_{s_j} (t, f), D_{s_i s_k} (t, f), D_{s_j s_k} (t, f) \]

cohercet auto and cross Cohen-class transforms of the described signals, then we can estimate the modal shapes as amplitude ratios, as follows:

\[
AR_{i,j} (t, f) = \sqrt{\frac{D_{s_i} (t, f)}{D_{s_j} (t, f)}}
\]

Or, better:

\[
AR_{i,j} (t, f) = \frac{D_{s_i s_k} (t, f)}{D_{s_j s_k} (t, f)}
\]
The work done by our group

A Nonlinear System Identification Method Based upon the Time-varying Trend of the Instantaneous Vibration Frequency and Amplitude

Volterra series applied to t-f distribution core

System Identification of Bilinear Systems through Separating Responses and Application to the Detection of Breathing Crack

Nonlinear damage detection using chaotic metrics such as Lyapunov Exponent, Correlation dimension
Non OMA identification for non-linear response

Review on Nonlinear System Identification

Model-free methods

- Methods for identifying the time-domain input-output relationship
- Methods for identifying the frequency-domain input-output relationship

Model-dependent methods

- Methods based on an extremely general model
- Methods based on a polynomial model
- Methods based on an equivalent linearization model
- Methods for time-varying systems
- Methods based on an exact model
Symptomatic and time-frequency techniques to non-linear structural identification
Non-linear behavior types

- **Hysteretic and plastic non-linearity** Evolving properties, energy dissipation, strength and stiffness increasing degradation: parametric identification: e.g. based on the inverse Bouc-Wen model assessment (The Bouc-Wen model does not respect fully some basic physical energy principles, so it should be used very carefully)

- **Elastic non-linearity, not evolving**, often generated by local damages, cracks, soft contact problems, anolonomous restrains.
Detection of elastic non-linearity: euristic approach

Time domain Hilbert transform

\[
H[y(t)] = \tilde{y}(t) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{y(\tau)}{t-\tau} d\tau
\]

Analytical signal

\[
Y(t) = y(t) + j\tilde{y}(t)
\]

Free decay analytical signal for linear behaviour
Free decay analytical signal for non-linear behaviour

Each kind of elastic non-linear behavior has a different “signature” on the complex plane

Invariant moments of free decay plots and correct prediction probability

\[ U(p, q) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (x - \mu_x)^p (y - \mu_y)^q f(x, y) dx dy \]
\[ \mu_x = \frac{M(1,0)}{M(0,0)} \quad \mu_y = \frac{M(0,1)}{M(0,0)} \]

Two 3-layers NNs for two stages: invariant moments fill the input nodes

**network 1; 26+10+6 nodes:** identification of the presence and type of non-linearity

**network 2; 52+15+3 nodes:** quantification of non-linearity
NN-1 outcomes: examples from simulated experiments

The invariant moment distribution for different non-linearity types

The probability of correct prediction by a trained Neural Network for a simple simulated case
NN-2 outcomes: examples from simulated experiments
Detection and simulation of an elastic non-linearity: approach by Volterra expansions

**Linear Identification**
- Frequency domain
- Time domain
- *Time-Frequency domain*

Time-Frequency domain identification holds in case of non-linear identification too.

**Key-fact:**
*Time-localized ("instantaneous") frequency domain analysis*
D’Alembert equation for a SDOF system

\[ m\ddot{y} + f_d(y) + f_s(y) = x(t) \]

- \( m \) is the mass, \( f_d(y) \) is the nonlinear damping term and \( f_s(y) \) is the nonlinear elastic restoring force

- Let’s assume \( f_d(y) \) and \( f_s(y) \) as:
  - polynomial functions of \( y \)
  - Depending from few unknown parameters

- \( y \) expandable in Volterra series
Volterra series representation in time domain

\[ y(t) = y_1(t) + y_2(t) + y_3(t) + \ldots \]
\[ y_1(t) = \int_{-\infty}^{+\infty} h_1(\tau_1 \cdot x(t-\tau_1) \cdot d\tau_1 \]
\[ y_2(t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h_2(\tau_1, \tau_2 \cdot x(t-\tau_1) \cdot x(t-\tau_2) \cdot d\tau_1 \cdot d\tau_2 \]
\[ y_3(t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h_3(\tau_1, \tau_2, \tau_3 \cdot x(t-\tau_1) \cdot x(t-\tau_2) \cdot x(t-\tau_3) \cdot d\tau_1 \cdot d\tau_2 \cdot d\tau_3 \]

**Instantaneous identification**

Applying the definition of the STFT to the Volterra series expansion:

\[ D(t, f) = \int_{-\infty}^{+\infty} y(\tau)w(\tau-t)e^{i2\pi ft} d\tau = \]
\[ = \int_{-\infty}^{+\infty} (y_1(\tau) + y_2(\tau) + y_3(\tau) + \ldots)w(\tau-t)e^{i2\pi ft} d\tau = \]
\[ = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h_1(\tau_1 \cdot x(\tau-\tau_1) \cdot d\tau_1 \cdot w(\tau-t)e^{i2\pi ft} d\tau + \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h_2(\tau_1, \tau_2 \cdot x(\tau-\tau_1) \cdot x(\tau-\tau_2) \cdot \cdot d\tau_1 \cdot d\tau_2 \cdot w(\tau-t)e^{i2\pi ft} d\tau + \]
\[ + \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h_3(\tau_1, \tau_2, \tau_3 \cdot x(\tau-\tau_1) \cdot x(\tau-\tau_2) \cdot x(\tau-\tau_3) \cdot \cdot d\tau_1 \cdot d\tau_2 \cdot d\tau_3 \cdot w(\tau-t)e^{i2\pi ft} d\tau + \ldots \]

For a Volterra system there must be convergence conditions to guarantee that the description is meaningful, involving generally bounds on the time interval and the input

If an analytical form for functions \( h_1, h_2, h_3 \ldots \) can be hypothesized, then it is possible to resort to a parametric method.

\( W(\tau-t) \) is a time-domain window function
Identification of Volterra systems

- Kernels are estimated through statistical moments of spectra obtained directly from experimental time series.

- In structural engineering applications it is not possible to obtain a number of experimental measurements as large as necessary to estimate the statistical quantities of interest especially when the dynamic tests are conducted in situ.

- The availability of a limited number of experimental measurements can be obviated by taking into account the “localisation” in time of the frequency content introducing a time-frequency description of the signals themselves.
Instantaneous identification

**BASIC ASSUMPTIONS**

- System’s response measured in N instants
- A vector \( \mathbf{p} \) of unknown parameters governs damping and elastic restoring forces

The parameters \( \mathbf{p} \) can be identified by convenient optimization procedures

\[
F_{ob}(n^*, \mathbf{p}) = \sum_{m=0}^{N-1} \left| D(n^*, m) \right|^2 - \left| D_s(n^*, m) \right|^2
\]

Objective function to minimize

\( F_{ob} \) describes the difference between the instantaneous energy of the signal at time \( t^* = n^*Dt \) and the instantaneous energy of the signal corresponding to a given configuration of the unknown parameters \( \mathbf{p} \), given by a Volterra series approximation.
Case study

Instantaneous identification of rigid bodies on nonlinear support based on Volterra series representation
The following procedure applies to the case, frequent in practice, of relatively soft and dissipating contact between upper block and lower non-linear elastic support. In such case the contact surface can be reduced during motion, but dynamic rebound effect are not present and the static push-over definition of the constitutive is well fit to the dynamic case too.

The following procedure does not generally apply to the case of hard and low dissipating contact between upper block and lower support. In such case the dynamic rebound and sudden change in rotation centre requires a more accurate cinematic model.
Instantaneous identification of a block structure

A non-linear structure made of two blocks is considered.

The blocks interacts one to each other via a contact surfaces without strength in traction.

Boundary conditions make possible only the relative rotation of the upper block (non-linear inverse pendulum).
Instantaneous identification of a block structure

As loads increase opening can happen and the structure exhibit a non-linear behaviour in the force-displacement law.
Instantaneous identification of a block structure

To check the results of identification, the force-displacement law has been approximated via a cubic polynomial:

\[ k(x) = k_1 x + k_3 x^3 = 8.193 \cdot 10^6 x - 3.673 \cdot 10^{12} x^3 \]
Instantaneous identification of a block structure

External load

System’s response
Instantaneous identification of a block structure

Seismic ground acceleration

System’s response
Instantaneous identification of a block structure

The instantaneous estimators of $\zeta$ and $f$ are characterised by a certain stability over the exact value, while estimator of $k_3$ is much more unsettled.

The mean value of $k_3$ is quite different from the exact value, but it is sufficient to reach a better model of the system's dynamic.
Reliability and hazard as stochastic process

- **Reliability of a structure** $R(t)$
  - *Probability that the time to reach a reference limit state, $t_b$, is greater than a given time $t$:

  $$R(t) = P(t \leq t_b)$$

- **Hazard function** $h(t)$
  - *Instantaneous rate of reliability deterioration:

  $$h(t) = \lim_{\Delta t \to 0} \frac{P(t_b < t + \Delta t \mid t_b \geq t)}{\Delta t} ; \quad R(t) = \exp \left( - \int_0^t h(x) dx \right)$$
Jumping into the symptom space

\[ R(S) = P(S \leq S_b \mid S = \text{suitable value}) = \int_{S}^{\infty} f_S \, dS \]

This formulation includes continuous time (slow degradation) or/and discrete time processes (earthquakes, storms etc.), given that time and symptom evolution can be correlated by suitable lows.
Condition Monitoring is essentially a search for structural or material disease symptoms.

- Symptoms can be regarded as evolutionary and sudden changes in observable qualitative properties and/or measurable responses.
- Symptoms search can require a knowledge based direct search or model based predictive assessment. In both cases a stochastic procedure is needed.
- In some applications direct search and model based simulations can provide an integrated procedure.
The Holy-Shroud Chapel case

- Designed by Guarino Guarini, built from 1667 to 1694
- Heavily damaged by fire in 1997
The numerical model
Thermal Analysis (fire effect simulation)

Deformed state after fire

Stress concentrations
(confirmed by direct observation)
Damage scenario

- Damage scenario = state of structure caused by an expected damage configuration

SOME KNOWLEDGE BASED DAMAGE SCENARIOS ARE DEFINED IN DETERMINISTIC WAY;

MANY OTHERS ARE THEN RANDOMLY GENERATED THROUGH STOCHASTIC CHANGES IN MECHANICAL PARAMETERS

EACH DAMAGE SCENARIO DSk PRODUCES A SET OF SENSITIVE SYMPTOMS

Symptom vectors are generated
From Damage Scenarios to Symptom vectors

Damage Indexes Vector

Symptom Observation Matrix

$DS_k$

$S(k,1)$ $S(k,2)$ $S(k,3)$ $S(k,N)$
From Symptom vectors to Damage Scenarios

- INVERSE APPROACH (ILL CONDITIONED)
  - Sensitivity-oriented problem-size reduction needed (Cempel’s approach)

Principal Component search → Equivalent actions based on SVD
Proper Orthogonal Decomposition
**DIRECT APPROACH**

- Many models are compared to select the best-fitting-one (Ian Smith)

### Symptom Observation Matrix

<table>
<thead>
<tr>
<th>$S(k,1)$</th>
<th>$S(k,2)$</th>
<th>$S(k,3)$</th>
<th>...</th>
<th>$S(k,N)$</th>
<th>...</th>
</tr>
</thead>
</table>

### MODELS

<table>
<thead>
<tr>
<th>$M_k$</th>
<th>...</th>
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</table>
Modal Identification after the main vibration test campaign (TFIE approach)
modal frequencies are marked by the downward peaks

X direction

Y direction
First two modal shapes

First mode

Second mode
Objective (Target) function:
- As usually, a (quadratic) function based on average error between the computed and measured output parameters.

Process organized in two phases
1. Multi-model generation, pre-selection and grouping;
2. Best fitting model final selection.
Model Updating in progress. Variable selected in a preliminary tentative process:

- 3D Livello02-2 (Variable S19)
- Tamburo Esterno (Variable S11)
- Timpani (Variable S6)
- 3D Livello 03 (Variable S18)
- Tamburo Interno (Variable S12)
PHASE 1

Probabilistic Global Search Lausanne (PGSL)

ITERATIVE PROCESS INCLUDING FOUR ENCASED LOOPS.

STARTING UP

- Each model is a point in $\mathbb{R}^N$ Input Variables Space (N floating variable input parameters);
- A set of M points are randomly generated
- Each point has the same joint probability density
- The range of each coordinate of each point is divided into $p$ equal intervals with the same probability
SECOND STEP
Adaptive probabilistic new models generation

Conceptual core:
SEARCH FOR A BETTER MODEL NEAR TO A GOOD-ONE

- Extract the best model (target function minimum)
- **Increase** the probability density in the range interval of each coordinate about the point representing the best model; **decrease** the probability density in the other intervals, as much as the distance from the best point is large.

Functionally similar to simulated annealing
THIRD STEP
Refining the search and generating new models

- Intervals with the highest probability density parted into equal sub-intervals
- Probability density distribution refined
- New models generation driven by the updated probability distribution
- Iterations to fulfill pre-defined criteria
PHASE 2
Evaluating models

**STEP 1: preliminary filtering**

- Discarding all models exceeding a "penalty" threshold defined on base of target and additional constraints.
STEP 2: Reducing the size of models and problem:

- Proper Orthogonal Decomposition (or SVD) on linear combinations of the input parameters allows to reduce the size of the space in which the models are described.
- The influence of the input parameters on the Target function can be assessed by few principal coordinates.
STEP 3: Reducing the number of models

- Clustering (K-means technique) makes it possible to collect the surviving models into groups.
- Models are associated to the cluster having the centroid at the minimum Euclidean distance.
- Due to the association of a new model, the centroid location evolves gradually.
Clustering

**INITIALIZING:** defining $k$ initial centroids

**ASSIGNING:** each representative point is assigned to the cluster of the nearest centroid

**RELOCATING CENTROIDS:** new centroids generated

**CHECKING:** $|c_{\text{new}} - c_{\text{old}}| < \text{tolerance}$ OR maximum number of iterations reached?

- **NO**
- **YES**

S**TOP**
The output parameters of the 5 centroids

Now we can proceed to finally select the best fitting model (minimum target value)

Or we can associate to each centroid a probability level related to the target value (bayesian estimator)
Conclusions

- The multi-model approach (Smith, EFPL) is a direct approach to model updating through a generation-selection of models. It makes it dual of the inverse model updating and somehow comparable to a GA approach.

+ Not ill conditionned
  - Stochastically oriented, potentially usable as a Bayesian approach.
  - Potentially robust

= No reliable updates if initial basic models not fit

- Reliability of results strongly influenced by the choice of the objective function. Wrong choice of the OF can cause ambiguous outcomes and numerical noise,
  - These problems can be avoided building the OF initially with few highly sensitive parameters and then assessing the effect of the other parameters in a hierarchical iterative procedure.
Conclusions

- Largely uncertain data require distributed sensing. On-line dynamic testing is mainly ambient testing, requiring robust identification techniques and redundant information.
- Redundant information requires low-cost sensing systems, with reduced needing of energy supply and protection from electric spikes and magnetic fields. Optical sensors and MEMS fulfill such needing.
- Among others, special sensors are under study as part of hierarchical sensor sub-systems to be placed in noisy locations.

- THEN, THANK YOU FOR YOUR PATIENCE!