ENERGY-BASED NON LINEAR STATIC ANALYSIS

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SUMMARY

The seismic behaviour of a structure can be analysed using non-linear static analyses (pushover analyses). The new Italian code introduces this procedure as an alternative design method. The applications require that the complex non-linear response of a MDOF model stressed by the increasing action of an assigned distribution of lateral forces is transformed in the response of a SDOF system, in order to compare it with an assigned design spectrum. The energy approach EA proposed in this paper is a rational improvement of the pushover analysis method devises by the first author of this paper; it can be used to carry out this transformation through a simple energy equivalence, so the comparison can be directly performed. According to the energy criterion of the Performance-Based-Design a suitable virtual energy equivalent displacement is then defined. It can be used to estimate a SDOF capacity curve which reproduces the total elastic and plastic energy cumulated by the whole structure during the pushing procedure. The use of this proposed procedure is illustrated and is compared with the methods indicated by the design literature, mainly when spatial problems are examined. The possibility of a design approach based on the use of energy spectra is also discussed.

1. INTRODUCTION

“Performance Based Seismic Design” (PBSD) and “Capacity Design” (CD) have moved attention from the concept of resistance towards the more general of performance evaluated in function of energy considerations, they represent the most significant aspect of recent progress in Seismic Engineering. This has been taken into account by the [Ordinanza PCM 3274,2003] (O3274) and the [Ordinanza PCM 3431,2005] (O3431) which updated the Italian code for the design of buildings in seismic zones, harmonizing them with the prescriptions of [Eurocode 8] (EC8). This has brought around a long waited substantial change in the concepts on which the previous codes based themselves. The new codes, in line with the most advanced criteria’s which today regulate the discipline, set the goal of amplifying the limits posed by previous design procedures so as to pursue more ambiguous objectives. On one hand the simple criteria of directly assuming as the demand a reduced earthquake, defined assigning an intensity independent of the potential ultimate performance (inelastic deformability and dissipative capacity) of each particular structural system, and without adequately correlating it to the local ground characteristics, has been abandoned. On the other hand new design criteria have been introduced which go beyond the ideas that verifying independently the resistance of each section, directly assuming as the local demand the stresses produced by an elastic study, using the forces produced by this reduced earthquake, without conditioning the effectiveness of post-elastic design and the potential collapses mechanisms. These new criteria has made it necessary to abandon the wide spread use, by designers, of the admissible tensions method for verification, which no longer respond to the new and more correct approach to the seismic safety problem.

Apart from opening up the road to the use of alternative seismic protection systems [Dolce, 2005] capable of reducing the energy transferred to the construction (seismic isolation and dissipation systems, not dealt with in

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The previous code), both the O3274 and O3431 orientate decisively the design towards the control of the post-elastic structural behaviour, applying the CD principles. The main objective is now moved towards the search for a general response capacity, even sustaining damage, against severe earthquake attacks which can succeed those frequently expected for the site. It is therefore necessary to control the residual inelastic response that succeeded the elastic limits of the materials, so limiting the risk of dangerous collapses which could put in danger the safety of people. By consequence, the new seismic code focuses the design attention on the control of the dissipative capacity and the ductility of the potential post-elastic mechanisms which could become mobilised by a seismic event of considerable intensity.

The seismic intensity taken as a reference to evaluate the limit performance is now indicated as the event which has a 10% probability of being exceeded in 50 years, corresponding to an average return period of 475 years. The seismic input is conventionally assigned through an elastic acceleration spectrum $S_e(T)$ corresponding to a viscous damping of 5%, taking into account the characteristics of the soil.

## 2. NON LINEAR ANALYSES OF THE NEW ITALIAN CODE

The principles of the CD and PBSD approaches base there validity on the control of the dissipative capacity associated with the post elastic deformability of the structure. In alternative to the traditional design procedures, the new code foresees that seismic design can be carried out using numerical analyses in an inelastic field, with the aim of confronting directly the global dissipative capacity of the potential collapse mechanisms with the seismic demand of the limit design event.

The direct method, consisting of the integration of the equations of motion applied to a three-dimensional inelastic model, conducts one to laborious numerical analyses. The procedure is not appropriate if not in the study of very particular situations. On the other hand, being impossible to perform accurate evaluation of the response to violent earthquakes, sophisticated design procedures are not necessary. And so both the Eurocode 8 and the O.3431 foresee the possibility to use the procedures of non linear static analyses ("pushover" analyses). This subject is discussed in this paper, proposing the use of an Energy Approach (EA) based only on evaluations of a energy nature [Parducci, 2004a and b].

## 3. THE USE OF THE ENERGY APPROACH IN THE PERFORMANCE BASED SEISMIC DESIGN

According to Eurocode 8, the Italian code uses the N2 method to carry out the PBSD. The PBSD is codified also by ATC-40 and FEMA-273-274 using other analogous numerical methods. The application procedures of these methods have been analyzed in some reports [Chopra & Goel, 1999; Fajifar, 2000; Priestley, 2001], which suggested the improvement of the EA method.

Since the PBSD fundamentally depends on energy concepts, the EA method uses only energy concepts, keeping away from any analytical step based on the elastic references of the modal shapes used in the other above-mentioned methods. As a matter of fact, every reference to the shape of an elastic mode can loose part of its significance when the design of the building is based on the performance of high dissipating systems, mainly when the dissipation mode modifies the elastic stiffness matrix of the structure, as in the case the elasto-plastic behaviour, that has been dealt with in this paper.

The purpose of the pushover analysis method is achieving a synthetic representation of the global seismic response of a MDOF (Multi Degree Of Freedom) structural system through the performance capacity curve of a simplified equivalent SDOF (Single Degree Of Freedom) model. The capacity curve of the SDOF model is defined in such a way that it can be used in the PBSD to compare it with the coherent acceleration spectrum of the maximum design earthquake, when the latter has been modified taking into account the effective damping capacity at the ultimate deformations. The comparison can be easily made if both the capacity and the demand are drawn in an ADRS (Acceleration Displacement Response Spectrum) representation.

The effectiveness of the PBSD procedure essentially depends on the rational choices concerning the following simplifications:

a) selection of the shape adopted to distribute the static horizontal forces acting on a MDOF model with which the mechanical characteristics of the structure have been reproduced;
b) selection of the reference displacement adopted to synthesize the global response of the MDOF model;
c) selection of the equivalence criterion adopted to define the performance curve of the equivalent SDOF model by means of the previous reference displacement;
d) selection of the criterion adopted to establish the coherence between the performance curve and the design spectrum, reduced from the elastic design spectrum, taking the actual damping capacity of the inelastic SDOF model into account.

Each of these choices leads to corresponding calculation steps. The final performance point PP is defined in the ADRS representation by the intersection of the design spectrum with the performance curve when the former has been made coherent with the latter, as Figure 1 shows. Coherence is obtained by modifying the design spectrum and taking into account the actual hysteretic dissipation capacity of the SDOF model, pushed just to the displacement of the performance point.

[Figure 1: Synthesis of the PP definition in ADRS plane]

The shape used to define the distribution of the $F_i$ forces in step a) is important, whereas the possible choices of practical interest are not very different, since they do not modify too much the numerical results [Fajifar 2000]. At the same time, as the Italian code suggests, two different distributions are generally used: one is proportional to the storey masses, another to the product of the storey masses multiplied by the displacements of the first modal shape.

The reference displacement of the MDOF model considered in step b) is generally chosen as the $d_{top}$ displacement of the top floor. Since this is an arbitrary choice, the $F^*$ force and $m^*$ mass that can be used in defining the equivalent SDOF model must be adapted consequently. The shape of the first elastic mode, normalized to the $d_{top}$ displacement, is generally used to perform this step. In the case of a planar structure, if $\Phi$ is the normalized vector of the modal shape and $I^*=(\Sigma \Phi m)/(\Sigma \Phi m^2)$ the corresponding participation factor, the force $F^*=F*/I^*$, the equivalent displacement $d^*=(\Sigma d)/I^*$ and the mass $m^*=(\Sigma m)/I^*$ are then assumed.

With regard to steps c) and d), as against the previous steps, the equivalence criteria are defined on the basis of energy concepts, according to the fundamental idea of the PBSD procedure. Particularly it occurs in the last step d), which is the fundamental step of the procedure. As matter of fact, the comparison of the performance curve $f^*=f^*(d^*)$ of the specific force $F^*=F*/m^*$ and the acceleration spectrum $S_\alpha^*=S_\alpha^*(d^*)$ requires that the two curves are coherent. The coherence is obtained by settling the acceleration spectrum as required by the actual dissipation capacity of the SDOF model (that is to say of the MDOF model) when it is stressed just to the maximum displacement $d^*$ of the performance point (Figure 1). The N2 method (Fajifar 1999) or an iterative procedure (ATC-40) can be performed to do this step.

In the first two steps the first modal shape is generally assumed because it is considered as the best representation of the lateral displacements of the MDOF model, although the fundamental criterion that makes the PBSD effective is in the following steps c) and d), which are based on energy considerations concerning the inelastic deformations. This approximation seems to be one of the reasons that have induced the Italian code to limit the application of the pushover analysis to regular structures. Other analogous limitations are indicated also by [ATC-40, 1996], and [FEMA-273, 1997].
The purpose of the EA is then to evade the conceptual approximation deriving from the use of elastic references to perform some of the steps concerning a numerical procedure that is essentially governed by energy concepts. Therefore, it defines a virtual performance displacement \( \delta^* \) on the basis of an energy equivalence.

4. THE ENERGY APPROACH

The EA avoids having to perform steps a) and b) of the pushover analyses by using an elastic modal shape as reference deformation criterion. Renouncing to assume a particular point of the MDOF model, it simply defines a virtual \( \delta^* \) displacement equalizing the energy deformations of both the MDOF and SDOF models.

In each \( k \) step of the pushover analysis the horizontal forces \( F^* \) applied to the floors \( i \) of the MDOF model grow with increments \( \Delta F_i \) which are proportional to the shape of the assigned force distribution. When the floor forces grow from \( (F^*_i)_k \) to \( (F^*_i)_{k+1} + (\Delta F^*_i)_k \) the floor displacements grow from \( (d^*_i)_k \) to \( (d^*_i)_{k+1} + (\Delta d^*_i)_k \), \( (\Delta d^*_i)_k \) being the incremental displacements imposed to the MDOF model by the \( k \) elastic step (or in the inelastic step if plastic deformations occur) which are calculated by performing a static analysis.

The force \( (F^*_i)_k = \sum_i (F^*_i)_k \) and the mass \( m^* = \sum_i m_i \) are adopted in the EA to define the behaviour of the energy equivalent SDOF model. The virtual displacement \( \delta^* \) that makes the latter energy equivalent to that of the MDOF model at each linear \( k \) step is then defined by the following incremental correlation (in Figure 2 it has been simplified):

\[
(F^*_i + \frac{1}{2} \Delta F^*_i)_k (\Delta \delta^*_i)_k = \sum_i \left[ (F^*_i) + \frac{1}{2} (\Delta F^*_i)_k \right] (\Delta d^*_i)_k
\]

(1)

Since the \( (\Delta d^*_i)_k \) incremental displacements are those produced by the \( (F^*_i)_k \) force at the \( k \) step, the \( (\Delta \delta^*_i)_k \) increment can be calculated:

\[
(\Delta \delta^*_i)_k = \sum_i \left[ (F^*_i) + \frac{1}{2} (\Delta F^*_i)_k \right] \frac{(\Delta d^*_i)_k}{(F^*_i + \frac{1}{2} \Delta F^*_i)_k}
\]

(2)

and the total \( \delta^* \) equivalent displacement reached at step \( k \) is obtained by adding all these incremental values:

\[
(\delta^*)_k = \sum_k (\Delta \delta^*_i)_k
\]

(3)

In the EA the behaviour of the energy equivalent SDOF model (that is to say of the performance curve), is then defined by the \( F^* \) force and \( \delta^* \) displacement:

\[
F^* = F^*(\delta^*)
\]

(4)

The \( \delta^* \) displacement does not correspond to a particular point of the MDOF model, but it is the virtual value which equalizes the energy capacities of both the models when the total shear force \( F^* = \sum F_i \) has been adapted in the SDOF model. It must be underlined that when the EA procedure is applied, the \( II(\delta^*) \) area of the performance curve \( F^*(\delta^*) \) reproduces exactly the deformation energy of the MDOF model.

Finally, introducing the specific response force \( f^* = F^*/m^* \), the specific performance curve \( f^*(\delta^*) \) can be drawn in the ADRS representation to be directly compared with its coherent acceleration demand (Figure 2) and the usual spectrum modification methods can be applied.
The performance curves \( F^*(\delta^*) \) or \( f^*(\delta^*) \) are generally transformed in bi-linear elasto-plastic curves, having a uniform plastic threshold. In this case, the elastic limit \( \delta_y \) is generally defined by maintaining the same area of the curves. If the EA has been used, the common correlation

\[
\Pi^* = \frac{1}{2} f^* \left( 2 \delta^* - \delta_y^* \right)
\]

between the deformation energy and the ductility ratio \( \mu = \delta^*/\delta_y^* \) assumes an actual significance. So, the structural analyses could be better performed by means of the design spectra defined with assigned ductility.

5. FORCE DISTRIBUTION FOR THE TRIDIMENSIONAL MODELS

Using the EA procedure results in that only static analyses and energy considerations are required. As regards the force distribution to be applied to the MDOF model in the first step of the pushover analyses, the EA suggests further solutions, alternative to those defined when referring to modal shapes. For example, the shape of the static deformation corresponding to the static storey displacement produced by forces proportional to the storey masses can be assumed. Since the forces are applied to the centre of the masses of each storey, the mechanical work required to define the equivalent \( \delta^* \) displacement can be easily calculated also whenever the torsion deformations are important.

The possibility of obtaining a performance curve that reproduces exactly both the elastic and plastic energies paves also the way to the use of energy spectra in seismic design.
6. USE OF ENERGY SPECTRA

Integrating the curve \( F(\delta) \) obtained in the way previously described, the deformation energy curve \( E(\delta) \) progressively accumulated in the MDOF model during the iterative process (Figure 3) can be obtained. In this way, also the dissipated energy is reproduced as a function of the virtual displacement \( \delta_\text{v} \). The function \( E(\delta) \) can also be calculated up to the displacement \( \delta_\mu \) which is associated to an assigned ultimate state of the structure. This ultimate state corresponds to a particular point of the equivalent bilinear elasto-plastic curve (which cannot be represented in the same graph), of the equal-energy response of the SDOF system. So, we can determine in the same way the \( \mu_\text{in} \) coefficient, which is representative of the global ductility of the structural system.

The possibility to obtain a capacity curve \( E=E(\delta) \) which reproduces in a direct way the inelastic deformation energy of the MDOF model, is associated with the idea of outlining a seismic design procedure assuming energy spectra as input. Therefore, the PBSD procedure, especially when it is applied to the seismic response of ductile structures, bases its validity on correlations of an energetic nature. In any case, it would seem more meaningful to make evaluations that compare the energetic capacity directly to the energetic demand instead of using evaluations based on the pseudo-acceleration spectra. The problem is discussed in [Mezzi, 2006]. Moreover, the capacity curves of ductile structures, that are prolonged in the field of the inelastic deformations, can be easily schematised in a bilinear representation that underlines a significant value of the ductility factor \( \mu \).

7. REFERENCES

Ordinanza PCM 3274/03 (2003), Primi Elementi in Materia di Criteri Generali per la Classificazione Sismica del Territorio Nazionale e di Normative Tecniche per le Costruzioni in Zona Sismica (in Italian).
Ordinanza PCM 3431/05 (2005), Ulteriori modifiche ed integrazioni alla Ordinanza PCM 3274 Primi Elementi in Materia di Criteri Generali per la Classificazione Sismica del Territorio Nazionale e di Normative Tecniche per le Costruzioni in Zona Sismica (in Italian).
Fajifar, P. (2000), Structural Analysis in Earthquake Engineering - A Breakthrough of Simplified Non Linear Methods, 12th ECEE, Elsevier Science Ltd.