Methods for the seismic analysis of transverse section of circular tunnels in soft ground

Méthodes pour l'analyse séismique de la section transversale des tunnels circulaires en terre

Emilio Bilotta, Giovanni Lanzano, Gianpiero Russo
DIG, University of Napoli Federico II, bilotta@unina.it, g.lanzano@unina.it, pierrusso@unina.it
Filippo Santucci de Magistris
SAVA, Engineering & Environmental Div., University of Molise, filippo.santucci@unimol.it
Francesco Silvestri
DDS, University of Calabria, f.silvestri@unical.it

ABSTRACT

The load increments induced by an earthquake in a tunnel lining can be ascribed to two different motion components: the ovalisation of the transversal section, induced by soil shear straining in the vertical plane, and the asynchronous movement of the ground-lining system along the longitudinal axis of the tunnel.

Forces induced by earthquakes in a tunnel lining can be assessed with several procedures at different levels of complexity. In this paper, the different levels of analysis are developed on idealised geometry and soil conditions, considered representative of soil classes specified by EC8 and the new draft Italian seismic code. Equivalent linear analyses along the transversal direction were carried out following two different design approaches, corresponding to increasing levels of complexity of analytical models, soil characterisation and description of seismic input:

- pseudo-static analysis, where the seismic input is reduced to an equivalent inertia force or a peak strain amplitude, computed through a free-field pseudo-static analysis of the ground and then considered acting on the tunnel lining in static conditions as well;
- full dynamic analysis, where the soil and tunnel responses are mechanically coupled and analysed via numerical modelling, such as finite element methods.

The sample problem is a circular tunnel of 6 m diameter, with the axis at a depth of 15 m in a 30 m thick layer of medium dense gravel, sand or soft clay, overlying a relatively stiff bedrock.

The force increments as calculated with the different procedures in the transverse section have been compared, showing a fair approximation to the full dynamic analysis achieved with the pseudo-static approach.

RÉSUMÉ

Les incréments de charge induits par un tremblement de terre dans un revêtement de tunnel peuvent être attribués à deux composants différents de mouvement: l'ovalisation de la section transversale, induite par le cisaillement du sol dans le plan vertical, et le mouvement asynchrone du système de sol-revêtement le long de l'axe longitudinal du tunnel. Ces forces peuvent être évaluées avec plusieurs procédures à différents niveaux de complexité. En cet article, les différents niveaux d'analyse sont développés sur une géométrie idéalisée et conditions de sous-sol considérées représentant des classes de sol indiquées par EC8 et le nouveau code séismique italien. Des analyses linéaires équivalentes le long de la direction transversale ont été effectuées après deux approches de conception différentes, correspondant à différents niveaux de complexité des modèles analytiques, de la caractérisation du sol et de la description du signal séismique:

- analyse pseudo-statique, où le signal séismique est réduit à une force équivalente d'inertie ou à une amplitude maximale de contrainte, calculée par une analyse pseudo-statique du mouvement séismique à champ libre et puis considérée agir sur le revêtement du tunnel en conditions statiques.
- analyse dynamique complète, où les réponses du sol et du tunnel sont mécaniquement couplées et analysées par modélisation numérique, comme les méthodes d'éléments finis.

Le problème analysé est d’un tunnel circulaire de diamètre de 6 m, avec l'axe à une profondeur de 15 m dans une couche épaisse 30 m de gravier moyen dense, de sable ou d'argile molle, recouvrant une roche en place relativement raide. Les incréments de force calculés avec les différentes procédures dans la section transversale ont été comparés, montrant que l'approche pseudo-statique réalise une juste approximation avec l’analyse dynamique.

Keywords: underground structures, tunnels, seismic loads, pseudo-static analysis, dynamic analysis
1 INTRODUCTION

It was recently understood that a sustainable development of urban areas requires an increasingly large use of underground facilities, especially to ensure a greater transportation demand, preserving the environmental quality. On the other hand, it was long-time understood that, in seismic areas, civil infrastructures and lifelines should be designed to support the extra loading produced by earthquakes. In spite of the above, no provisions are given in EC8 (EN 1998-1, 2003) on how to evaluate seismic loading on underground structures. On the other hand, some indication can be found in Owen & Scholl (1981), JSCE (1992), AFPS/AFTES Guidelines (2001), ISO TC 98 (2003). To bridge this gap some research activities are in progress in Italy, aimed at developing reliable methods for the analysis of the behaviour of underground structures and lifelines should be designed to support the extra loading produced by earthquakes. In spite of the above, no provisions are given in EC8 (EN 1998-1, 2003). This paper describes the most recent results achieved in this research project.

During an earthquake, ovaling or racking deformations of a transverse section of a tunnel are mostly due to shear waves propagating perpendicularly to the tunnel axis, resulting in a distortion of the cross-section of the structure. In this paper, forces induced by ground shaking in the tunnel lining are evaluated using several calculation procedures at different levels of complexity, which fall into the class of the pseudo-static and dynamic methods.

The analytical simulations have been performed on three idealized ground conditions (Fig. 1): a 30 m thick layer of soft clay, medium dense sand or gravel, overlying a soft rock half-space as a bedrock (V_s = 800 m/s, γ = 22 kN/m^2, D_0 = 0.5%). The tunnel has the following characteristics:

- circular shape with reinforced concrete lining (variable thickness, diameter D=6 m);
- axis depth z_0=15 m, length 1000 m.

![Figure 1. Ground conditions](image)

The values of small strain soil parameters have been chosen by means of literature empirical relations, linking the shear modulus (G_0) and the damping ratio (D_0) to the lithostatic stress, the void ratio and intrinsic soil properties, such as particle size and plasticity index I_P (Santucci de Magistris, 2005; d’Onofrio & Silvestri, 2001). Starting from G_0(z), it is possible to obtain the shear wave velocity profile V_s(z) as:

![Figure 2. Variation of shear modulus and damping with shear strain level](image)

V_s(z) = \frac{G_0(z)}{\rho}

where \rho is the soil density. The variations of V_s with depth for each soil model are shown in Fig. 1, where the dashed lines represent the value of the so called ‘equivalent velocity’ V_{s,30} (EN 1998-1, 2003). Table 1 summarizes the geotechnical parameters and the ground type according to EC8.

<table>
<thead>
<tr>
<th>Ground</th>
<th>type</th>
<th>\phi^* (°)</th>
<th>I_P (%)</th>
<th>\gamma (kN/m^2)</th>
<th>D_0 (%)</th>
<th>V_{s,30} (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>D</td>
<td>25</td>
<td>30</td>
<td>18</td>
<td>2.5</td>
<td>124</td>
</tr>
<tr>
<td>Sand</td>
<td>C</td>
<td>35</td>
<td>-</td>
<td>20</td>
<td>1.0</td>
<td>239</td>
</tr>
<tr>
<td>Gravel</td>
<td>B</td>
<td>44</td>
<td>-</td>
<td>21</td>
<td>1.0</td>
<td>401</td>
</tr>
</tbody>
</table>

To account for soil non-linearity and cyclic energy dissipation, in the computational models it is also necessary to introduce the variation of shear modulus G and damping ratio D with the shear strain level \gamma. Therefore, the curves G(\gamma)/G_0 and D(\gamma) for the three materials (Figure 2) have been assumed according to literature indications:

- for clay and sand, the curves suggested by Vucetic & Dobry (1991) for I_P = 30% and 0%;
- for gravel, the relationships reported by Stokoe (2004) for D_{50} = 10 mm.
ground-tunnel system, using a numerical method. On the other hand, in simplified methods the kinematic soil-structure interaction is neglected and free-field displacements are applied to the tunnel boundary (e.g., Hashash et al., 2001). Moreover, the effects of compression waves are also neglected, as only shear waves propagating in vertical planes might induce shear strain \( \gamma \). Thereafter, distortions are used to calculate seismic force increments in the tunnel lining by means of closed-form elastic solutions, such as those by Wang (1993) or Penzien (2000).

2.1 Pseudo-static analysis

Bilotta et al. (2007) discussed the results of four different methods developed in the framework of a pseudo-static analysis, to evaluate the maximum shear stress \( \tau_{\text{max}} \). They were all based on the equilibrium of a deformable soil column from the surface to a given depth \( z \).

A first class of methods (namely, method 1 and method 2 in the above referenced paper) needed to specify a vertical profile of peak acceleration \( a_{\text{max}} \) and the maximum shear stress \( \tau_{\text{max}} \) was computed by integration as:

\[
\tau_{\text{max}}(z) = \int_0^z \alpha a_{\text{max}}(z) \, dz
\]

Another class of methods (i.e. method 3 and method 4) followed an approach similar to the simplified procedures used in the evaluation of liquefaction susceptibility by reducing with depth the maximum value of surface acceleration \( a_{\text{max,s}} \). Hence the shear stress distribution with depth was calculated according to the following equation of dynamic equilibrium of a soil column:

\[
\tau_{\text{max}}(z) = r_d(z) \frac{a_{\text{max,s}} \sigma_v(z)}{g}
\]

In Eq. (3), \( \sigma_v \) is the total vertical stress, and \( r_d \) is a reduction parameter which takes into account the deformability of the soil column. This latter can be calculated as a function of depth \( z \), for instance through the relationships given by Iwasaki et al. (1978) and Liao & Whitman (1986), or including the effects of magnitude, as suggested by Idriss & Boulanger (2004). Further values of \( r_d(z) \) can be derived by the reduction factors of \( a_{\text{max,s}} \) with depth given by Power et al. (1996).

Seismic site response (SSR) analyses were also performed by Bilotta et al. (2007) with different input signals to evaluate an average function of the reduction factor with depth. The results showed that method 3, which used a reduction factor \( r_d = 1 - 0.015z \) (m) according to Iwasaki (1978), provided a fairly good agreement with the average results of the series of SSR analyses, at least for the subsoil profiles considered.

All the pseudo-static methods considered require a preliminary evaluation of the peak acceleration at surface; its value has been computed as:

\[
a_{\text{max,s}} = S \cdot a_g
\]

where \( a_g \) is the peak acceleration on outcropping rock site and \( S \) the site response factor. Its value has been either assumed constant, as specified by EC8 and its subsequent proposals of updating (i.e. Italian OPCM 3274, 2003; ETC12, 2006) or varying with the ground motion amplitude, as proposed by Ausilio et al. (2007). In the latter case, a non-linear response factor (FRN) was assumed, for each soil class, as a negative power law of \( a_g \) as reported in Table 2.

<table>
<thead>
<tr>
<th>Soil</th>
<th>EC 8</th>
<th>OPC M</th>
<th>ETC1 2</th>
<th>FRN (Ausilio et al., 2007)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>1.35</td>
<td>1.35</td>
<td>1.1</td>
<td>0.539 ( (a_g/g)^{0.4171} )</td>
</tr>
<tr>
<td>Sand</td>
<td>1.15</td>
<td>1.25</td>
<td>1.15</td>
<td>1.0624 ( (a_g/g)^{0.2362} )</td>
</tr>
<tr>
<td>Gravel</td>
<td>1.2</td>
<td>1.25</td>
<td>1.3</td>
<td>1.0177 ( (a_g/g)^{0.2017} )</td>
</tr>
</tbody>
</table>

The maximum shear strain at a depth \( z \) is therefore calculated from the maximum shear stress, \( \tau_{\text{max}}(z) \), according to the Ramberg & Osgood (1943) model:

\[
\gamma_{\text{max}}(z) = \frac{\tau_{\text{max}}(z)}{G_v} + C \left[ \frac{\tau_{\text{max}}(z)}{G_v} \right]^R
\]

where the parameters \( C \) and \( R \) have been calibrated on the curves of Fig. 2 (Valentino, 2006).

2.2 Dynamic analysis

A set of input acceleration time histories has been selected from a database of records of Italian seismic events (Scasserra et al., 2006). All the signals have been scaled to a value of \( a_g \) equal to 0.35g and applied at the base of the subsoil models.

The finite elements (FE) software Plaxis v8 (Brinkgreve, 2002) has been used to perform two-dimensional free-field and soil-structure interaction dynamic analyses. The code allows defining the damping tensor \([C]\) through the Rayleigh formulation, i.e. as a linear combination of the mass tensor \([M]\) and the stiffness tensor \([K]\):

\[
[C] = \alpha_g[M] + \beta_g[K]
\]

Coefficients \( \alpha_R \) and \( \beta_R \) have been calculated according to the double frequency method, assuming that the damping ratio is about constant between the first natural frequency of the deposit and a frequency \( n \) times larger; \( n \) is the first odd integer which ap-
proximates by excess the ratio between the fundamental frequency of the seismic signal and the first natural frequency of the deposit. This method avoids the overestimate of damping throughout the considered frequency range (Lanzo et al., 2004). The bedrock has been assumed as a rigid boundary, whereas lateral mesh boundaries, about 40 D aside from the tunnel, are modelled as dampers according to the Lysmer & Kuhlemeyer (1969) formulation (Fig. 3).

The FE analyses were performed with a linear elastic model for the soil. The dependency of the soil stiffness and damping ratio on the strain level has been first considered by equivalent linear analysis. Therefore, preliminary one-dimensional SSR analyses have been performed by means of the code EERA (Bardet et al., 2000), which operates in the frequency domain. Ground conditions and soil behaviour have been modelled according to Figs. 1 and 2. The material properties calculated as output from the SSR analysis were hence used as input to the FE analyses.

The methods based on the integration of the $a_{max}(z)$ profile (i.e. 1 and 2) give very close results and tend to overestimate the shear strain with respect to those based on the reduction of $a_{max,s}$ (i.e. 3 and 4). In the case of gravel (soil type B) the different site factors have similar values (being FRN=1.26), therefore the profiles corresponding to the same method are al-

most overlapped. This is relatively true for sand also: for soil class C, EC8 and ETC12 specify the same site response factor, smaller than OPCM 3274; for $a_s = 0.35g$, the non-linear response factor is larger (1.36). In the case of clay, OPCM 3274 and EC8 specify the same value, whereas ECT12 proposed a lower value and the FRN is even lower than unity (0.83) for stronger ground motions.

In all, the computed profiles of shear strain appear more dispersed and by using the non-linear response factor all the computed shear strains are at the lower bound of the cluster of the profiles computed with the same method. This last result should not to be intended as general: the non-linear response factor is in fact higher than $S$ for weaker motions (see Ausilio et al., 2007).

The shear strains calculated at the tunnel depth with the four pseudo-static methods, $\gamma_{ps}$, have been averaged for each soil and plotted in Fig. 5 against the corresponding values, $\gamma_{dyn}$, calculated by Plaxis for the same soil profiles in free-field conditions. The lines in the figure have been obtained by adding and subtracting the standard deviation to the mean values, plotted as markers. The solid black line represent the condition $\gamma_{PS} = \gamma_{dyn}$. It can be noted that only method 4 underestimates the shear strain computed by Plaxis. Furthermore, the scatter of data and the deviation of pseudo-static predictions from the results of dynamic analyses increase with soil deformability. On the whole, both for gravel and sand, method 3 and 4 give fairly consistent results, whereas for clay, the best prediction is by method 4.
Under the hypothesis of rough interface between the lining and the soil, the variation of hoop (N) and bending moment (M) with the angle $\theta$ is given by:

$$N(\theta) = \frac{1}{2} K_2 G_m D \gamma_{PS} \cos \left( \theta + \frac{\pi}{4} \right)$$  \hspace{1cm} (7a)

$$M(\theta) = \frac{1}{12} K_1 G_m D^2 \gamma_{PS} \cos \left( \theta + \frac{\pi}{4} \right)$$  \hspace{1cm} (7b)

where:

$$K_1 = \frac{12(1-v_m)}{2F+5-6v_m}$$  \hspace{1cm} (8a)

$$K_2 = 1 + \frac{H[(1-2\nu_m)-(1-2\nu_n)] - \frac{1}{2} (1-2\nu_n)^2 + 2}{H(3-2\nu_n)+(1-2\nu_n)[C - \frac{5}{2} - 8\nu_n + 6\nu_n^2] + 6 - 8\nu_n}$$  \hspace{1cm} (8b)

The dimensionless parameters:

$$F = \frac{G_m (1-v_f^2)D^3}{2E_f t^3}$$  \hspace{1cm} (9)

$$C = \frac{G_m (1-v_f^2)D}{E_f (1-2v_m)}$$  \hspace{1cm} (10)

represent the relative soil/tunnel stiffness, i.e. are inversely proportional to the influence of soil-structure interaction.

The maximum lining internal forces (i.e. at $\theta = \pi/4$) calculated by uncoupled soil–tunnel analyses can be normalised dividing eqns. 7a,b by $\gamma_{PS}$ in the following way:

$$N_{max,PS}/\gamma_{PS} = \frac{1}{2} K_2 G_m D$$  \hspace{1cm} (11a)

$$M_{max,PS}/\gamma_{PS} = \frac{1}{12} K_1 G_m D^2$$  \hspace{1cm} (11b)

In a similar way, the lining internal forces calculated in FE analyses, accounting for soil-tunnel interaction, can be divided by the shear strain calculated in dynamic free-field analyses, obtaining $N_{max,dyn}/\gamma_{dyn,ff}$ and $M_{max,dyn}/\gamma_{dyn,ff}$.

In Figs. 6a-b the average values (markers) and bands of deviation (lines) of normalised internal forces obtained for the case of sandy subsoil from the pseudo-static methods are plotted against those calculated from the FE analyses. The different data colours refer to a lining thickness increasing from 0.1 to 1.3 m. 

Figure 6. Comparison between dynamic and pseudo-static analyses in terms of hoop (a) and bending moment (b)

The normalised internal forces increase with the lining thickness, indicating the increasing effect of soil-structure interaction on the seismic load increment in the tunnel. Accordingly, the maximum bending moment and hoop computed by full dynamic analysis result increasingly higher than those obtained by the uncoupled pseudo-static approach, although the difference appears acceptable for flexible lining (up to about 30 cm). Among the pseudo-static procedures, method 4 again provides a better agreement with the FE analysis.

In Figs. 7a,b, the ratios between the maximum dynamic and pseudo-static bending moments and hoops computed for sand have been plotted against the lining thickness, t.
Note that, whatever the pseudo-static method adopted, beyond a threshold lining thickness such ratios does not depend on the value of t. Moreover, the maximum hoop computed by pseudo-static analyses, $N_{\text{max}}^{PS}$, always underestimates the corresponding maximum dynamic hoop, $N_{\text{max}}^{DYN}$.

4 FINAL REMARKS

To summarize, the following dimensionless parameters can be defined:

$$\alpha = \frac{\gamma_{PS}}{\gamma_{DYN,FF}}$$  \hspace{1cm} (12)

$$k_N = \frac{N_{\text{max}}^{PS}/\gamma_{PS}}{N_{\text{max}}^{DYN}/\gamma_{DYN,FF}}$$  \hspace{1cm} (13a)

$$k_M = \frac{M_{\text{max}}^{PS}/\gamma_{PS}}{M_{\text{max}}^{DYN}/\gamma_{DYN,FF}}$$  \hspace{1cm} (13b)

For the ratios plotted in Fig. 7 it follows:

$$\frac{N_{\text{max}}^{PS}}{N_{\text{max}}^{DYN}} = \alpha \cdot k_N$$  \hspace{1cm} (14a)

$$\frac{M_{\text{max}}^{PS}}{M_{\text{max}}^{DYN}} = \alpha \cdot k_M$$  \hspace{1cm} (14b)

The variable $\alpha$ accounts for the variation of strains calculated according to different pseudo-static methods. In Table 3 the values of $\alpha$ are shown, as computed by eq. (12) on the average shear strain of each method.

<table>
<thead>
<tr>
<th>Method</th>
<th>Gravel/Sand</th>
<th>Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.5</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>2.3</td>
<td>4.7</td>
</tr>
<tr>
<td>3</td>
<td>1.4</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>0.4</td>
<td>0.8</td>
</tr>
</tbody>
</table>

The values of $\alpha$ for sand and gravel are the same. Different is the case of clay, for which the average values are about twice as larger.

The ratios $k_M$ and $k_N$ are different for different pseudo-static methods, as they depend on the values of $G_m$ (i.e., on the shear strain level). In Figs. 8 and 9 the values $k_N^* = k_N/G_m^{PS}$ and $k_M^* = k_M/G_m^{PS}$ are plotted against the thickness $t$: note that $k_N^*$ seems substantially independent on the choice of pseudo-static method.

![Figure 8](image1.png)

Figure 8. Ratio $k_N^* = k_N/G_m^{PS}$ vs lining thickness

![Figure 9](image2.png)

Figure 9. Ratio $k_M^* = k_M/G_m^{PS}$ vs lining thickness
By means of any of the above mentioned pseudo-static methods, the following expressions may be used to evaluate the maximum bending moments and hoops, taking into account the possible kinematic interaction:

\[ N_{\text{max}}^{\text{DYN}} = \frac{N_{\text{max}}^{\text{PS}}}{\alpha \cdot k_N \cdot G_m^P} \]  
\[ M_{\text{max}}^{\text{DYN}} = \frac{M_{\text{max}}^{\text{PS}}}{\alpha \cdot k_M \cdot G_m^P} \]  

where \( N_{\text{max}}^{\text{PS}} / G_m^P \) and \( M_{\text{max}}^{\text{PS}} / G_m^P \) are computed by means of one of the pseudo-static methods (1 to 4) according to a straightforward rearrangement of (11a-b); \( \alpha \) follows for each method from Table 3; \( k_N^* \) and \( k_M^* \) can be obtained, for sand, from the charts in Figs. 10 and 11, for a given thickness \( t \).

5 ACKNOWLEDGMENTS

This work is a part of a Research Project funded by ReLUIS (Italian University Network of Seismic Engineering Laboratories) Consortium. The Authors wish to thank the coordinator, prof. Stefano Aversa, for his continuous support and the fruitful discussions. The strong motion database used in this study was developed as part of an ongoing joint project involving researchers from the University of Rome La Sapienza and the University of California, Los Angeles, with support from the Pacific Earthquake Engineering Research Center. Preliminary results from this group were reported by Scasserra et al. (2006), but the data utilized here have not been published.

6 REFERENCES

AFPS/AFTES, Guidelines on earthquake design and protection of underground structures, 2001
Brinkgreve R.B.J., Plaxis 2D version8. A.A. Balkema Publisher, Lisse, 2002
OPCM 3274, “Primi elementi in materia di criteri generali per la classificazione sismica del territorio nazionale e di normative tecniche per le costruzioni in zona sismica”, GU Repubblica Italiana, 105-8/5/03, 2003
Santucci de Magistris F., “Fattori di influenza sul comportamento meccanico dei terreni”, App. B in 'As-
petti geotecnici della progettazione in zona sismica - Linee Guida AGI', Associazione Geotecnia Italiana, Patron, Bologna, 2005 (in Italian)


JSCE - The Japanese Society of Civil Engineers, “Earthquake resistant design for civil engineering structures in Japan”, 1992

