Methods and design equations for flexural reinforcement of RC beams with pre-tensioned externally bonded FRP sheets

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Summary

It will be presented:

- an introduction summarizing advantages and disadvantages of prestressing the FRP
- a review of the current prestressing methods
- the main technological issues,
- the new tensioning device suitable for FRP fabrics,
- its concept and the design scheme of the device,
- a design procedure for prestressed FRP,
- some remarks of the system.

Introduction

Prestressing the FRP prior to bonding - general:
Advantages

- provides stiffer behaviour as at early stages
- greater structural efficiency: as the neutral axis lays at a higher depth in prestressed sections, most of the concrete is in compression
- cracks formation in the shear span is delayed, and when cracks appear they tend to be more finely distributed and narrower
- favours closure of pre-existing cracks, improving both serviceability and durability (reduced cracking),
- also enhances the shear resistance of members (larger uncracked concrete section contribute to shear resistance)

Introduction

Other Advantages of prestressing the strips prior to bonding

- the same amount of strengthening can be achieved with smaller areas of pre-stressed strips compared with unstressed strips,
- prestressing significantly increases the applied load at which mild steel yields, compared to non-stressed members
- if adequate anchorage is provided, prestressing may increase the ultimate resisting moment by avoiding failure modes associated with peeling-off, both at cracks and at the ends of the strips,

Introduction

Disadvantages of the technique:

- more expensive than normal strip bonding

  (greater number of operations and special equipment required)

- the operation takes somewhat longer,

  (the equipment that holds the strip to the beam soffit must remain in place until the adhesive has sufficiently hardened)

- when the prestressing force is too high, failure of the beam due to release of the prestressing force may occur at the two ends

  (result of the development of severe shear stresses in the concrete just above the FRP).

Introduction

Hence:

- design, detailing and execution of the end zones require special care.
- if no special anchorages are provided at the ends, FRP strips shear-off from the ends (with prestress levels in the order of only 5-6% of their tensile strength, for CFRP).

  But a rational prestress requires a degree of prestressing in the range of 50% of the FRP tensile strength, which may only be achieved through the use of special anchorages applying vertical confinement, or *other techniques*.
- Several systems have been developed for practical applications as well as research purposes.

Introduction

Prestressing of Fabrics:

Use of prestressed FRP fabrics has not increased as widely as that of laminated strips.

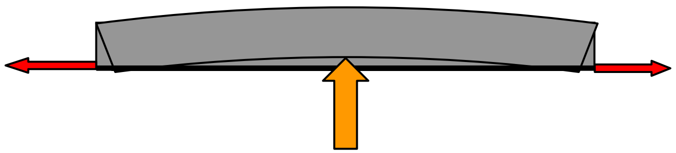
- due to the difficulty of seizing the fabric when prestressing it without ripping the fibres:
- devices effective in seizing and prestressing laminated strips have been produced,
- while *significant problems* have been encountered in developing as effective devices for FRP fabrics.

Quick review of current methods

- Two main methods of prestressing the external strengthening:

indirect or direct

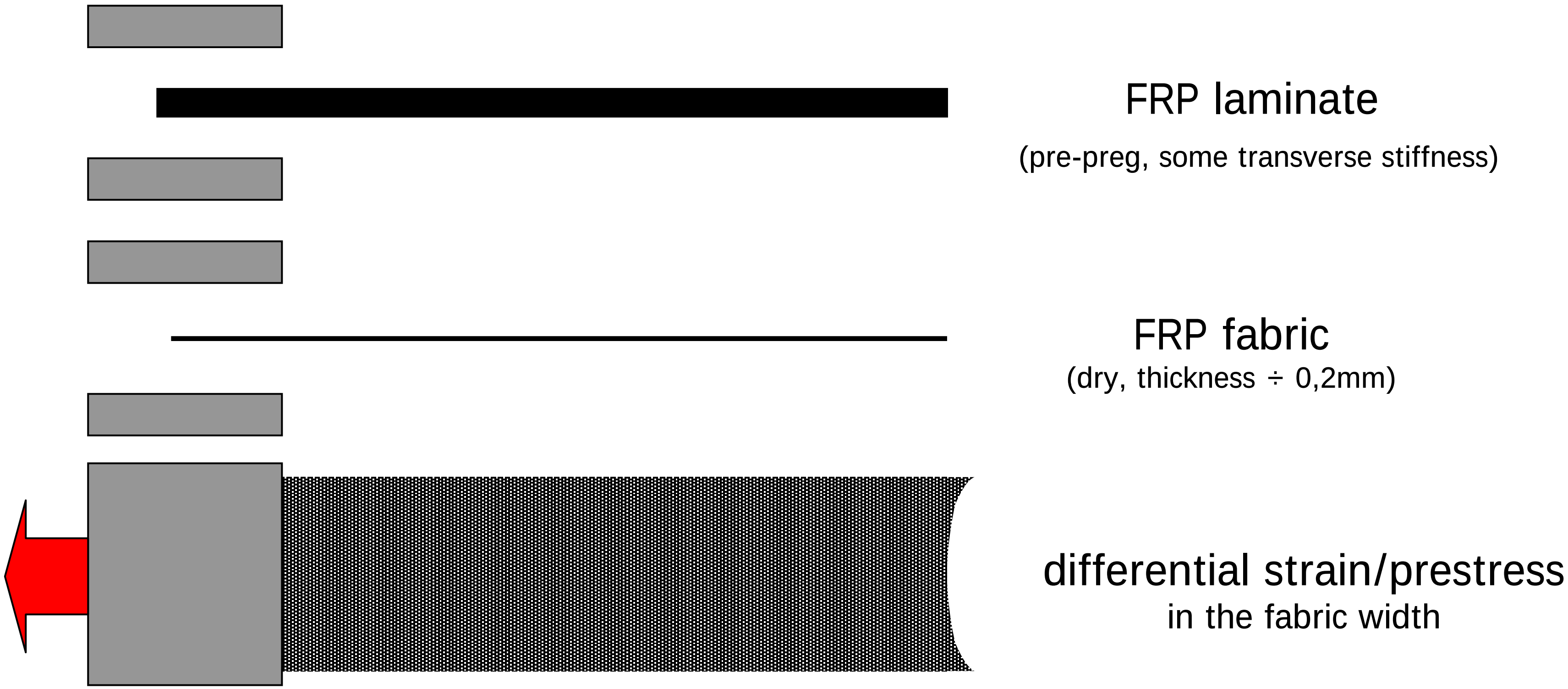
indirect: by cambering the flexural member



direct: jacking against an external reaction frame or on the beam itself

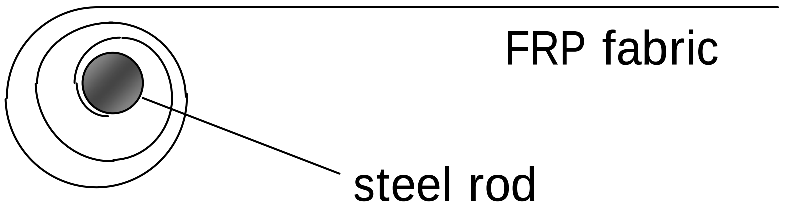
Technological issues

- The main technological challenge of prestressing FRP fabrics consists in seizing the fabric itself



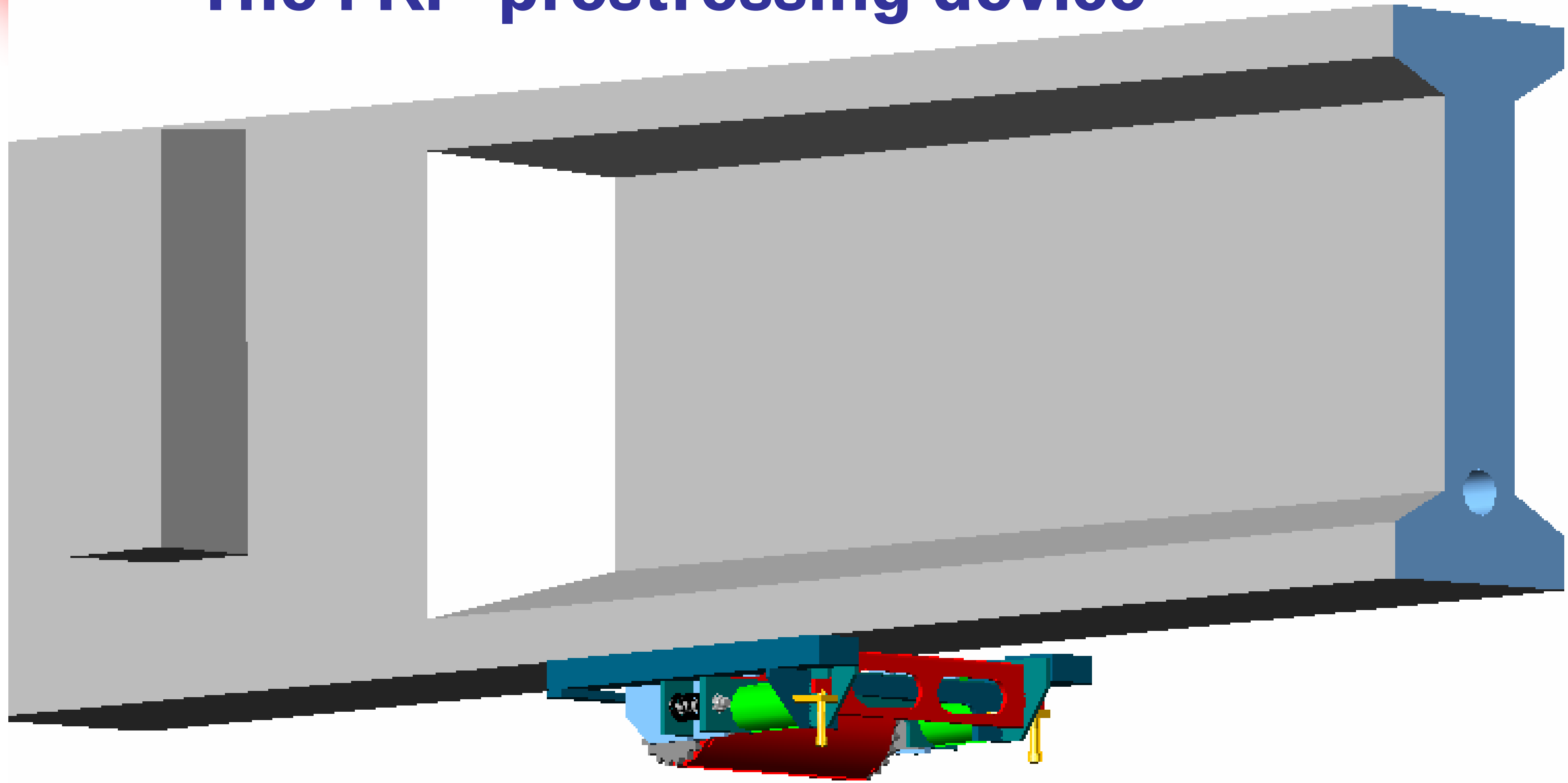
Technological issues

- So far the only reliable way to prestress fabrics is to wrap it several times around a resin-impregnated steel rod



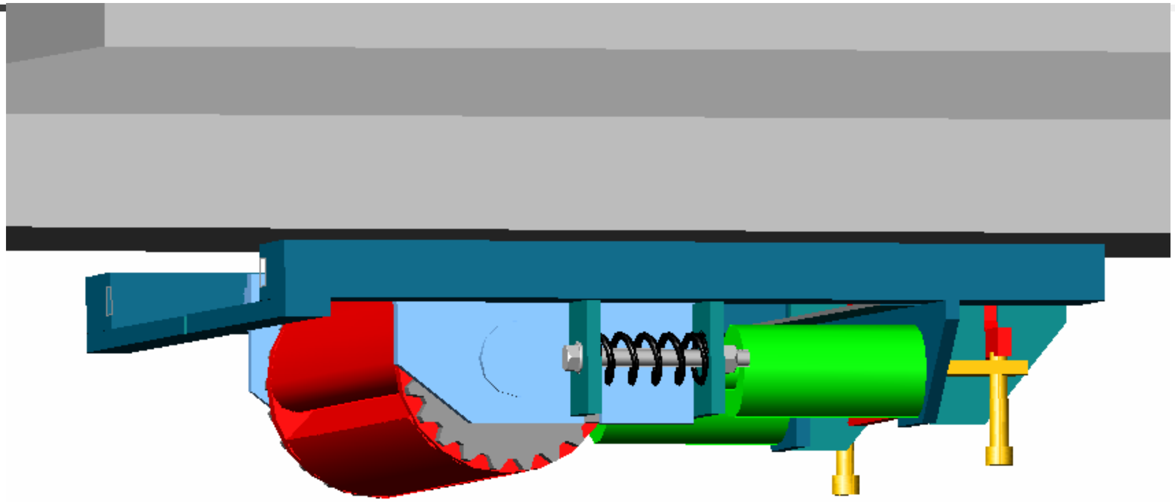
- Several applications exist, all of them need the time for the resin to cure before prestressing the fabric.
- The rod remains in place after the prestressing process

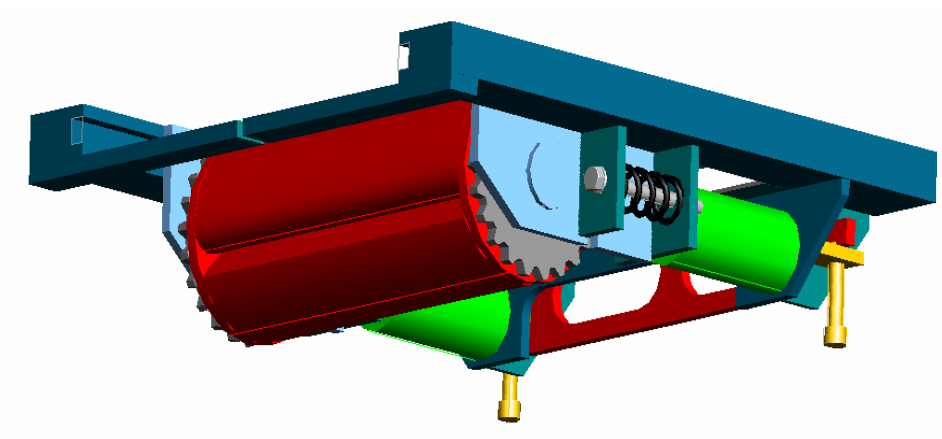
The FRP-prestressing device



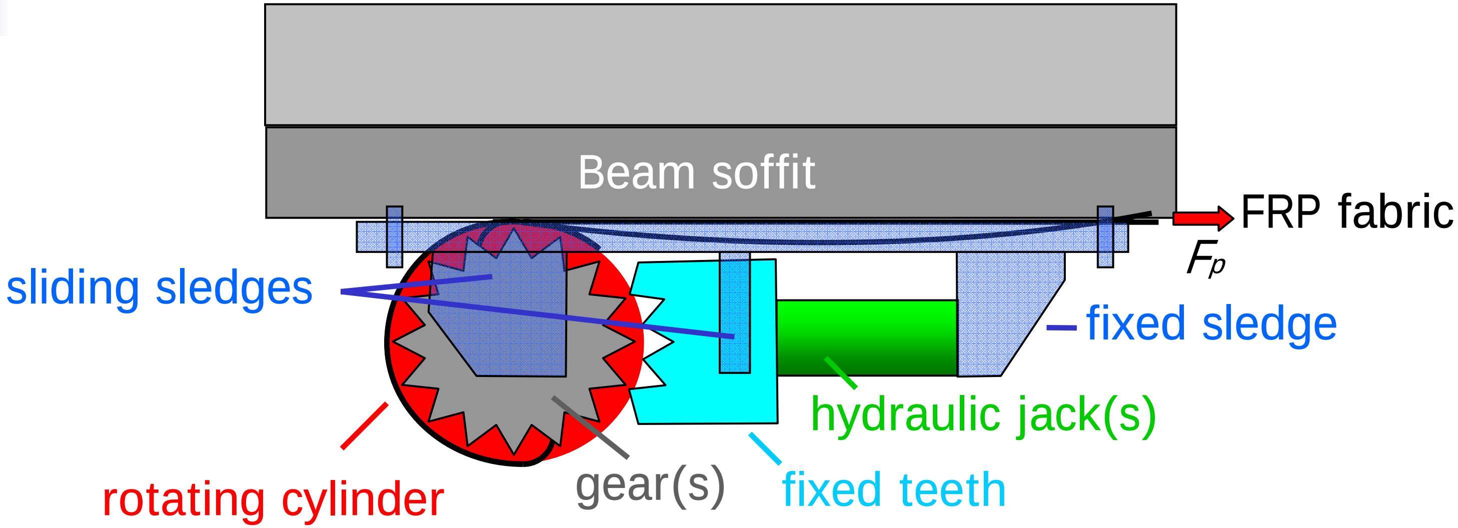
mounted under a bridge beam

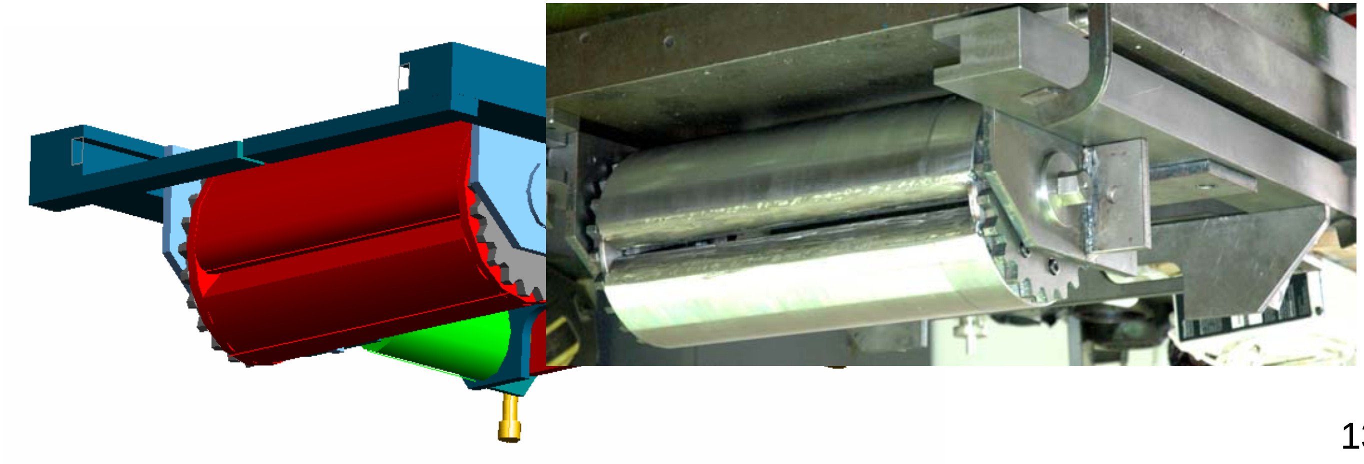
The FRP-prestressing device





The FRP-prestressing device

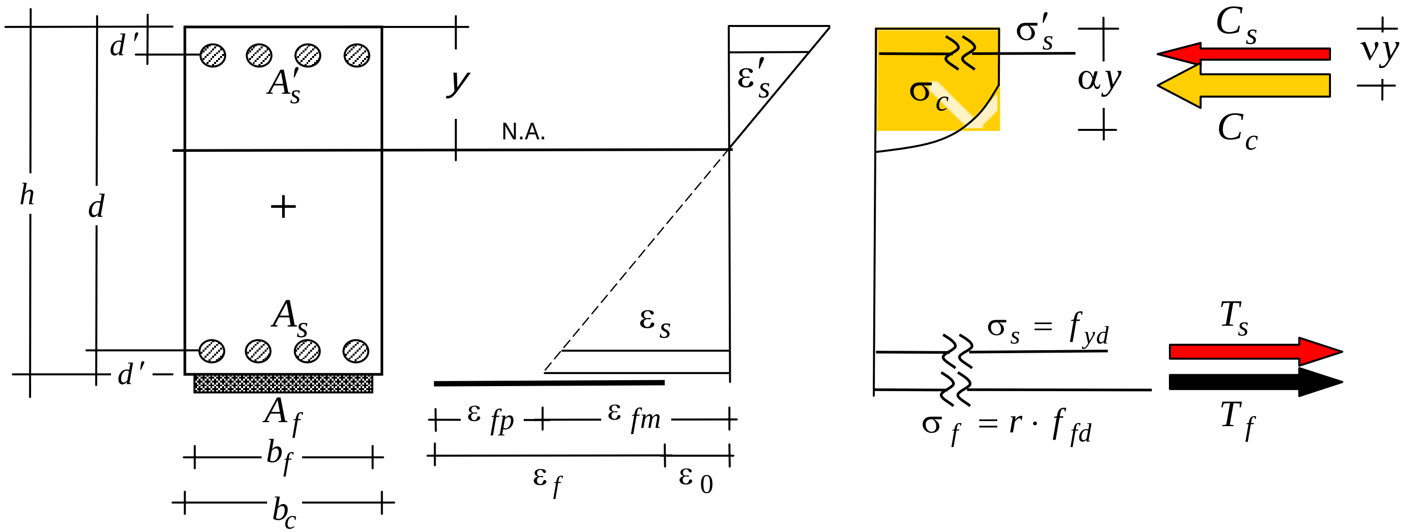




Design of flexural strengthening with prestressed FRP

- The case of a reinforced concrete beam with inadequate steel reinforcement is treated.
- The analytical developments follow the steps commonly taken:
  - Assess the resisting moment capacity of the beam,
  - Evaluate the difference with respect to the demand,
  - Design the required amount of strengthening material and its prestress level, *using closed-form equations*

Geometry and material properties



b_c = section width

h = section depth

d' = concrete cover thickness

d = h-d' = effective depth

A_s = tension steel area

A'_s = u A_s = compressive steel area

f_{ck} = concrete characteristic strength

$f_{cd} = \alpha_{cc} f_{ck}/\gamma_c$ = *concrete design strength*

$E_c = 22000(f_{cm}/10)^{0.3}$ = *concrete modulus*

$f_{cm} = f_{ck} + 8$ MPa = *concrete mean strength*

$\varepsilon_{cu} = 0.0035$ = *concrete crush strain*

f_{yk} = steel characteristic strength

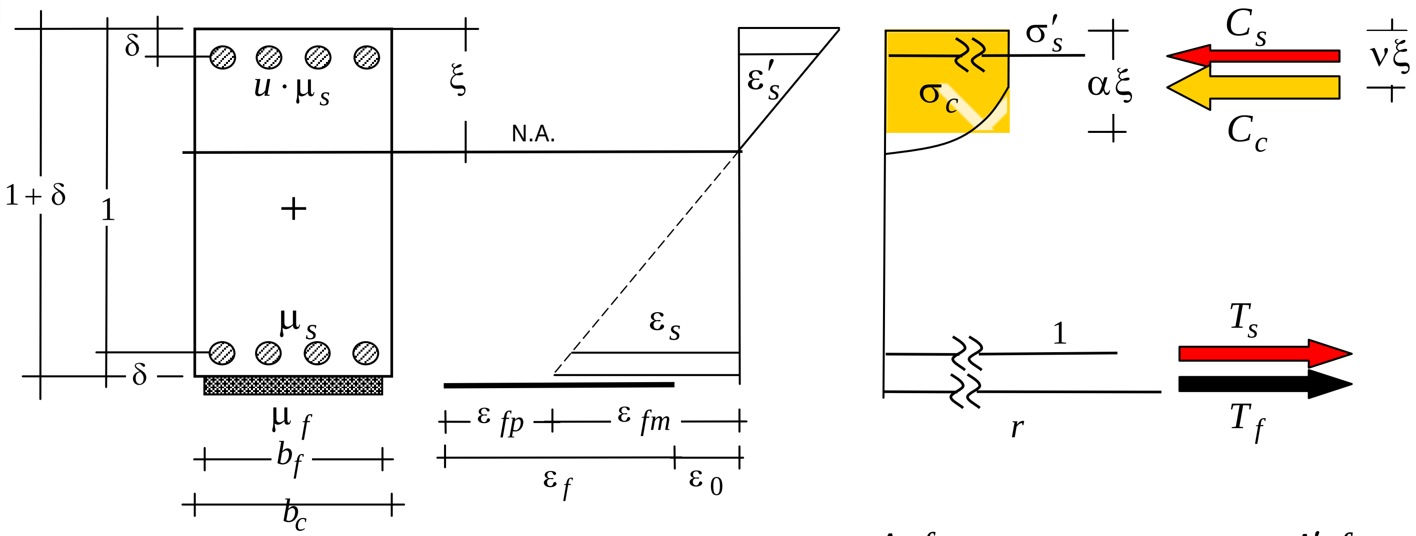
$f_{yd} = f_{yk}/\gamma_s$

= *steel design yield strength*

$\varepsilon_{yd} = f_{yd}/E_s$

= *steel design yield strain*

Non-dimensional formulation



$\xi = \frac{y}{d}$ = *neutral axis depth*

$\delta = \frac{d'}{d}$ = *concrete cover ratio*

$\eta = h/d = 1+\delta$ = *beam depth ratio*

u = A'_s / A_s = *compressive/tensile steel ratio*

w =1-*u* = *complementary steel ratio*

$\mu'_s = u\, \mu_s$ = *mechanical steel ratios*

$$\mu_s = \frac{A_s f_{yd}}{f_{cd}\, b_c\, d}$$

(compression steel)

$$\mu'_s = \frac{A'_s f_{yd}}{f_{cd}\, b_c\, d} = u \cdot \mu_s$$

(tensile steel)

A_f = *area of the FRP sheet/plate*

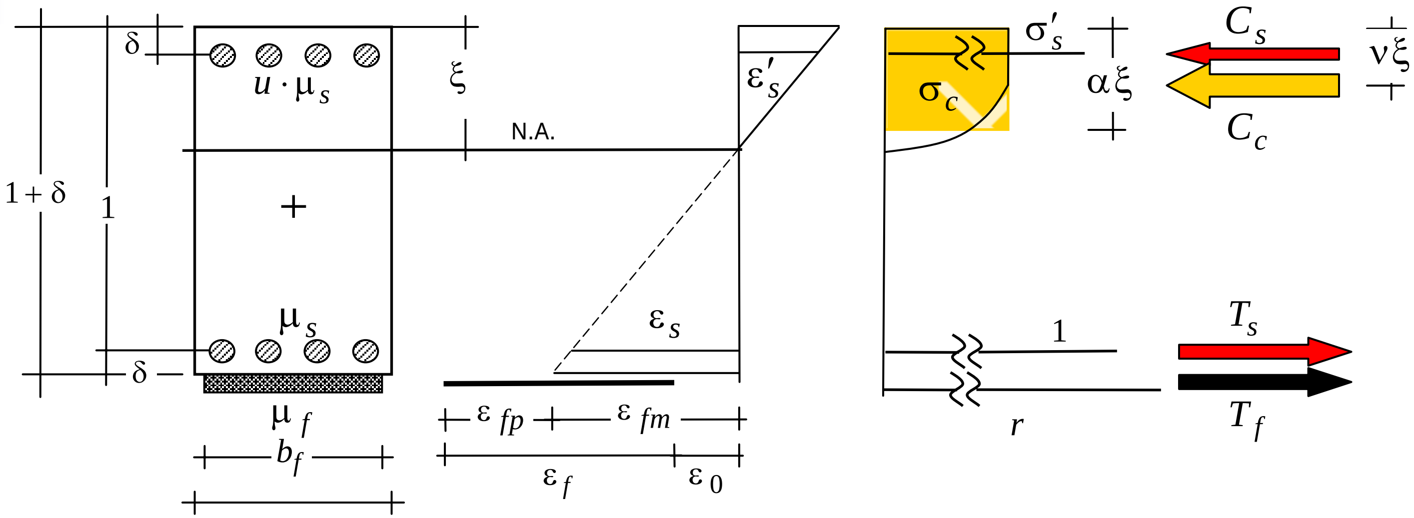
b_f = *sheet width*

t_f = *sheet thickness*

$f_{fd} = E_f\, \varepsilon_{fd}$ = *design FRP strength*

ε_{fd} = *design ultimate strain of FRP*

Non-dimensional formulation



- all moments can be normalized as:

$$m = \frac{M}{f_{cd}\, b_c\, d^2}$$

Compatibility

- Given that *section collapse* occurs when $\varepsilon_c = \varepsilon_{cu}$

- Through compatibility the corresponding *strains* at the tension and compression steel bars are easily given:

$$\varepsilon_s(\xi) = \varepsilon_{cu} \frac{d-y}{y} = \varepsilon_{cu}\left(\frac{1}{\xi} - 1\right) \qquad \text{(tension)}$$

$$\varepsilon'_s(\xi) = \varepsilon_{cu} \frac{y-d'}{y} = \varepsilon_{cu}\left(1 - \frac{\delta}{\xi}\right) \qquad \text{(compression)}$$

- while the *stress* in the compression steel is written in non-dimensional form as

$$s'(\xi) = \frac{\sigma'_s}{f_{yd}} = \min\left[\frac{\varepsilon_{cu}}{\varepsilon_{yd}}\left(1 - \frac{\delta}{\xi}\right), 1\right]$$

Condition assessment

- In non-dimensional form, the *existing* resisting moment is found by solving the following translation equilibrium equation for the neutral axis depth ξ

$$0 = \alpha \cdot \xi + s'(\xi) \cdot u \cdot \mu_s - \mu_s$$

where $\alpha = 0.8$ = concrete resultant coefficient

- The solution of the above equation is used to find the non-dimensional resisting moment, by replacing it in the following equation

$$m_{Rd} = s'(\xi) \cdot u \cdot \mu_s (\nu\ \xi - \delta) + \mu_s (1 - \nu\ \xi)$$

where $\nu = 0.4$ = concrete resultant centroid coefficient

Initial conditions before FRP-strengthening

- Before applying the FRP strengthening, it is necessary to *know the strain state at the beam soffit.*
- In case of propped beam, this is equal to zero
- If the beam is left under the characteristic (mean) value of the permanent load G_k under service conditions, the initial strain ε_0 is evaluated as

$$\varepsilon_0 = \min(\varepsilon_{0,cr}, \varepsilon_{0,id})$$

and can be found as:

$$\varepsilon_0 = \frac{M_{G_k}}{E_c \, I_{(cr \text{ or } id)}}(h - y)$$

FRP-strengthening design target

- The aim of the strengthening procedure through prestressed FRP is to comply with the requirement:

$$m_{Rd,f} = incr \cdot m_{Rd} \geq m_{Sd}$$

- The design must fulfil that the FRP strain does not exceed the design maximum strain, possibly reduced by a device efficiency (uncertainty) factor ***r***, based on the prestressing device used

$$\varepsilon_f(\xi) = \varepsilon_{fm}(\xi) + \varepsilon_{fp} \leq r \cdot \varepsilon_{fd}$$

where $\varepsilon_{fm}(\xi)$ = mechanical strain, found based on compatibility as

$$\varepsilon_{fm}(\xi) = \varepsilon_{cu}\left(\frac{\eta}{\xi} - 1\right) - \varepsilon_0$$

and ε_{fp} = prestress strain

Strengthening with prestressed FRP

- First step: choose the appropriate FRP material *only* on the basis of the *design ultimate strain* ε_{fd}

- The device efficiency (or uncertainty) factor can then be expressed as:

$$r = \frac{\varepsilon_{fm}(\xi)}{\varepsilon_{fd}} + p_f \leq 1$$

where $p_f = \frac{\varepsilon_{fp}}{\varepsilon_{fd}}$ is the FRP prestress ratio with respect to its ultimate strain

- The above quantity is the objective of the design,

- Together with the FRP-strengthening ratio

$$\mu_f = \frac{A_f \, f_{fd}}{f_{cd} \, b_c \, d}$$

where f_{fd} is the FRP design strength

Strengthening with prestressed FRP

- The sectional equilibrium equations are in this case:

$$0 = \alpha \cdot \xi + s'(\xi) \cdot u \cdot \mu_s - \mu_s - r \cdot \mu_f$$

$$m_{Rd,f} = s'(\xi) \cdot u \cdot \mu_s (\nu\ \xi - \delta) + \mu_s (1 - \nu\ \xi) + r \cdot \mu_f (\eta - \nu\ \xi)$$

- where:
  - the target value $m_{Rd,f}$ depends on the value chosen for the increment $incr$,
  - and $\eta = h/d = 1 + \delta$

- By solving the two above equations, together with the limitation given by the efficiency factor r, the three unknowns ξ , p_f, μ_f, are found

Design through closed-form equations

- Simple design equations can be obtained by making the assumption that the stress in the compressive steel is $s'(\xi) = 1$

  (low contribution from the yielded compressive steel)

- The two (translational and rotational) equilibrium equations then become

$$0 = \alpha \cdot \xi - w \cdot \mu_s - r \cdot \mu_f$$

$$m_{Rd,f} = \mu_s \cdot (1 - u \cdot \delta - w \cdot \nu\ \xi) + r \cdot \mu_f (\eta - \nu\ \xi)$$

- From the former equation the neutral axis depth is found

$$\xi = \frac{w\ \mu_s + r\ \mu_f}{\alpha}$$

$$\eta = h/d = 1 + \delta$$

$$u = A_s'/A_s$$

$$w = 1 - u$$

Design through closed-form equations

substituting which in the latter we obtain

$$m_{Rd,f} = \mu_s (1 - u\ \delta) + r\ \mu_f \eta - \frac{\nu}{\alpha}(w\ \mu_s + r\ \mu_f)^2$$

Then:

- the amount of FRP is found

$$\mu_f = \frac{1}{2\nu r}\left[\alpha \cdot \eta - 2\nu w \cdot \mu_s - \sqrt{\alpha^2 \cdot \eta^2 - 4\nu\ \alpha [m_{Rd,f} - \mu_s (u - \delta)]}\right]$$

- along with the prestress to be applied

$$p_f = r - \frac{\varepsilon_{cu}}{\varepsilon_{fd}}\left(\frac{\alpha \cdot \eta}{w\ \mu_s + r\ \mu_f} - 1\right) + \frac{\varepsilon_0}{\varepsilon_{fd}}$$

Computation of the amount of FRP

- Having determined the amount of FRP in non-dimensional terms, we find:

1): The FRP area

$$A_f = \frac{\mu_f \, f_{cd} \, b_c \, d}{f_{fd}}$$

by selecting the FRP design strength f_{fd}
and (implicitly) the FRP modulus $E_f = f_{fd} / \varepsilon_{fd}$

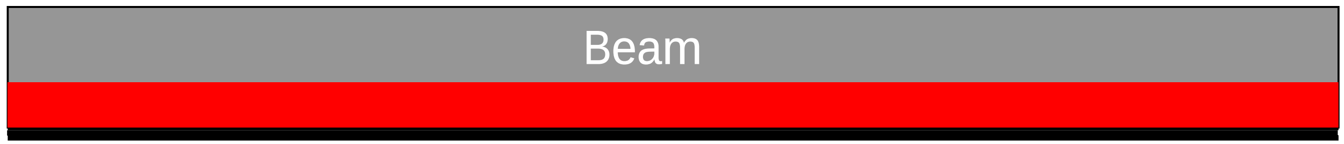
2): The applied prestress:

$$\sigma_{fp} = p_f \cdot r \cdot f_{fd}$$

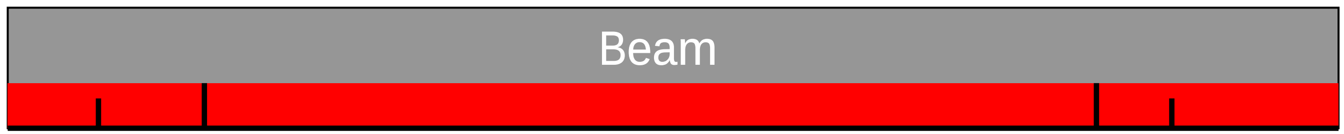
3): and the force to apply to the FRP by means of the prestressing device

$$P_{fp} = \sigma_{fp} \cdot A_f$$

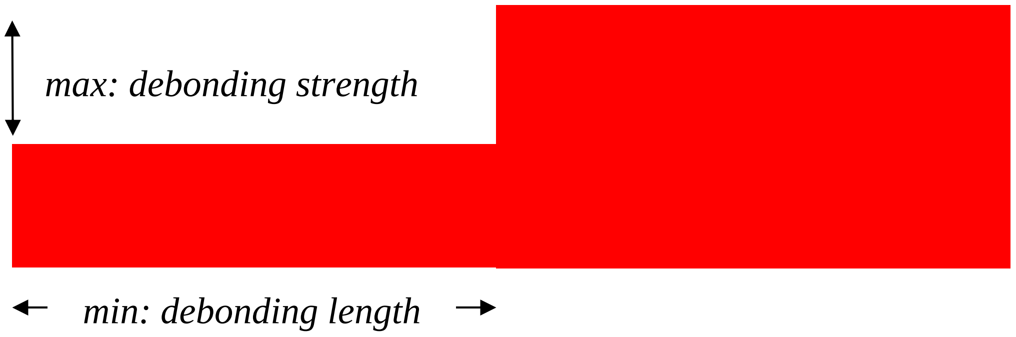
Remarks of the device



Prestress level in
the first layer



Prestress level in
the only layer



Conclusions

- A new device has been developed to properly seize and prestress with accuracy FRP layers in shape of fabrics.
- The device is easily handled, reusable and adjustable to the local work conditions and design needs.
- Prestress level and total stress in the fabric are continuously monitored and easily adjustable to the specific design needs.
- A design procedure of easy and immediate use for professionals is proposed.
  This allows to determine, making use of *closed-form equations*, the amount of FRP and the prestressing level that provide under-designed reinforced concrete sections with the required moment resistance increase

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Thank you!

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