ReLUIS WP3 Task 3.3 final (summary) report - Structural reliability and redundancy factor for bridge infrastructures

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Introduction

This report presents a concise summary of research activities conducted on system-based versus component-based reliability assessment for existing bridges. Although the findings primarily focus on existing structures, the methodologies and conclusions are equally applicable to new bridge designs. Addressing system-level reliability is of particular significance, as current design codes, such as NTC 2018 and Eurocodes, do not explicitly consider system-level reliability targets. This omission presents a critical challenge, especially for bridge infrastructure, where the interaction among structural components plays a key role in overall performance and safety.

The full set of results and in-depth analyses are provided in a comprehensive version of the report. This summary aims to highlight the key findings and methodologies, offering a streamlined overview for the reader. At the end of each section, key results are summarized for quick reference, allowing a faster reader to extract essential information efficiently.

*0.1. Context & Essential Literature Review***

The AASHTO 2016 standards introduced a redundancy factor, η_R, in component-level verifications to account for structural redundancy in system reliability assessments. This factor is incorporated within a broader safety coefficient, which also includes ductility and operational classification coefficients. The prescribed values of η_R are 1.05 for non-redundant members, 1.0 for conventional redundancy levels, and 0.95 for highly redundant systems. However, this approach is considered preliminary, with its application left to engineering judgment, and the standard itself acknowledges the need for further refinement.

Frangopol's study [1] provides the basis for a reliability-based calibration of η_R, using idealized system configurations and probabilistic parameters. Although still in its early stages, the AASHTO approach has been adopted and further developed in this project to align with its research objectives. The study aims to propose a more structured and systematic application of redundancy concepts in structural engineering practice.

0.2. Motivation and scope of this study

0.2.1. Motivation

The current Eurocode and NTC 2018 do not explicitly address structural system reliability. Structural design verifications are performed at the component level, focusing on individual failure modes (e.g., shear or bending) or single cross-sections. This approach creates a gap in codification, as system-level failure probabilities' often governed by interactions among multiple failure modes' are not directly considered.

It is widely assumed that redundancy and ductility enhance system reliability, resulting in higher reliability levels compared to individual members. However, this assumption breaks down in cases involving low-ductility behavior, such as brittle failures, and/or in structures with low redundancy, such as statically determinate systems. In such cases, the probability of system failure can exceed the failure probabilities of individual components. Consequently, structures may meet the code-target reliability at the component level but fail to achieve the same reliability at the system level. This "code paradox" applies to both new and existing structures.

0.2.2. Scope

To address these critical issues, this study proposes the introduction of a redundancy safety factor at the resistance level. This factor is designed to adjust component-level reliability targets to ensure alignment with system-level reliability requirements. The approach maintains compatibility with current practices, focusing on component-level verification while avoiding the complexities of explicit system reliability analysis. The structural systems considered in this study are primarily existing Italian bridges. Specifically, the research activities are centered on two main objectives:

Objectives

1. Calibration of the redundancy factor using a mechanical approach to structural system reliability.
2. Identification of a range of η_R values for potential application to existing bridges.

The first objective is theoretical, extending the work of Frangopol [1] and others by incorporating structural load distributions into system reliability analysis. The second objective is more practical, aimed at defining a framework for future implementation into current building standards, such as those outlined in AASHTO.

1. The redundancy factor

1.1. Redundancy factor definition

The aim of this section is to introduce the redundancy factor, following the work of Frangopol [1]. This is defined as a reliability-based partial safety factor, that aims to restore the system reliability, to the target value established by the reference building standards. We measure reliability by the generalized reliability index[1]; specifically, β_c refers to the component reliability index, β_s refers to the system reliability index, and $\bar{\beta}$ to the target reliability index.

A generic component limit state function is defined as:

$$g_c = g_c(R_c; P) = R_c - P \begin{cases} > 0 & \text{if the component survives} \\ \leq 0 & \text{if the component fails} \end{cases}, \tag{1}$$

where P is the random variable that represents the load of the component and R_c is the random variable that represents the resistance of the component. The two variables are statistically independent, and their distribution is assumed to be Gaussian or Lognormal. In this setting, the probability distributions of the load and resistance are fully characterized by the means $E[P], E[R_c]$, and their coefficients of variation $\delta[P], \delta[R_c]$. Notice, that it is

[1] $\beta = \Phi^{-1}(p_f)$ [2], being $\Phi^{-1}(\cdot)$ the inverse standard normal Cumulative Density Function and p_f the probability of failure

assumed equal distribution parameters for each component. The component reliability indices for the Gaussian and Lognormal cases are given by

$$\beta_c = \frac{E[R_c] - E[P]}{\sqrt{(E[R_c]\delta[R_c])^2 + (E[P]\delta[P])^2}}, \tag{2}$$

$$\beta_c = \frac{\ln\left(\frac{E[R_c]}{E[P]}\sqrt{\frac{1+\delta[P])^2}{1+\delta[R_c])^2}}\right)}{\sqrt{\ln\left(1+\delta[R_c])^2\right)\left(1+\delta[P])^2\right)}}. \tag{3}$$

Building codes (including NTC 2018 and Eurocodes) assign a given reliability class, corresponding to a specific target $\bar{\beta}$, and enforces verifications at the component level, i.e. $\beta_c \geq \bar{\beta}$. Given that, for *pre-assigned* $E[P], \delta[P]$, and $\delta[R_c]$, the $E[R_c]$ is derived from Eq. (2) and Eq. (3) by imposing $\beta_c = \bar{\beta}$ as shown in Fig. 1.

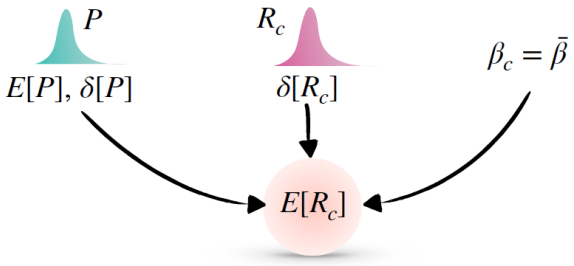

Figure 1: Component-wise approach defined by current building standards

Following this procedure, we can define the *ideal compliant building code design* as the design for which each component has the prescribes reliability index target. However, this design does not ensure that β_s meets that same target value $\bar{\beta}$, as it does not account for the components' configuration within the structural system. To address this problem it is necessary to enforce $\bar{\beta}$ at a system level.

The assumptions regarding the load and resistance distributions remain consistent with previous considerations. Additionally, it is assumed that the loads on all components are comonotonic[2]. For Gaussian and Lognormal

[2]This assumes a common external load, which is redistributed among components according to deterministic mechanical rules.

cases, this implies perfect correlation between the loads (i.e., $\rho[P_i, P_j] = 1$). The resistances of all components are also assumed to share a uniform correlation coefficient, denoted by $\rho[R_i, R_j] = \rho$. Under these assumptions, and for a given system limit state function, the structural system reliability depends on the variable $E[R_c]$ and is expressed as $\beta_s(E[R_c])$. To ensure the target reliability is achieved at the system level, the condition $\beta_s(E[R_c]_s) = \bar{\beta}$ is enforced, where $E[R_c]_s$ denotes the mean component resistance that satisfies the system-level reliability requirement (see Figure 2).

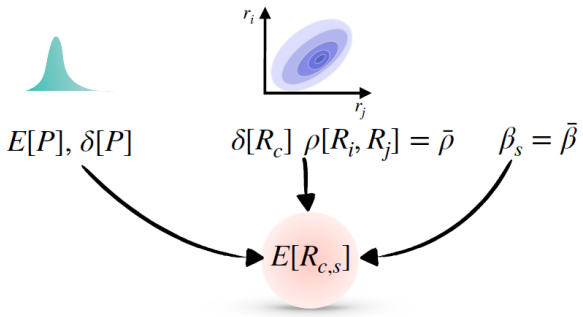

Figure 2: System-wise approach hereby introduced

The redundancy factor is then defined as the ratio between the mean component resistance provided by the component-wise design procedure $E[R_c]$, and the mean component resistance provided by the system-wise design $E[R_c]_s$, i.e.

$$\eta_R = \frac{E[R_c]_s}{E[R_c]}. \tag{4}$$

The redundancy factor η_R can be then applied during the component-wise design procedure, according to two distinct approaches:

$$\text{to the resistance:} \qquad \sum_i \gamma_i Q_i \leq \frac{R_i}{\gamma_m \eta_R}, \tag{5}$$

$$\text{to the load:} \qquad \sum_i \gamma_i Q_i \eta_R \leq \frac{R_i}{\gamma_m}. \tag{6}$$

In this study, the coefficient is applied at the resistance level, as this approach aligns with the understanding that redundancy is an inherent property of structures. This choice is consistent with the logic of current building codes,

which differentiate between brittle and ductile failures (e.g., see the NTC 2018 verification for tensile forces in the presence of bolts).

Observe that when non-redundant and/or brittle members are present, $\eta_R > 1$. This implies that the mean component resistance is reduced, ensuring that compliance with the safety check requires a higher component reliability: $\beta_C \geq \bar{\beta}$. The objective in this case is to restore the system reliability, β_s, to meet the target reliability, $\bar{\beta}$, thereby adhering to the underlying philosophy of the regulation. Conversely, for redundant or ductile members, $\eta_R < 1$. Here, the mean component resistance is increased, ensuring that compliance with the safety check requires a lower component reliability: $\beta_C \leq \bar{\beta}$. This procedure can be utilized when the designer aims to take advantage of the system's reserve reliability. This approach is not recommended for new structures, where both component-level and system-level target reliabilities should be satisfied. However, for existing structures, the system's safety reserve can potentially be utilized in specific verifications. The explicit application of this coefficient in such cases is beyond the scope of the present study.

1.2. Calibration of redundancy factor

This section focuses on the computation of the redundancy factor η_R. The assumptions regarding components load and resistance probability models are the ones described in Section 1.2. At first the following input parameters are given:

- $E[P]$: mean value of the external load distribution.
- $\delta[P]$: coefficient of variation of the external load distribution.
- $\delta[R_C]$: coefficient of variation of the component resistance distribution.
- ρ: correlation index between the i-th and j-th components resistances.
- System limit state function, $g_s(\cdot)$, expressed as a function of component limit state functions.

As previously described, restoring the target system reliability necessitates adjustments to the mean component resistance, $E[R_c]$, which is treated as the sole variable. The procedure for determining the reliability-based redundancy factor follows the algorithm outlined in Figure 3.

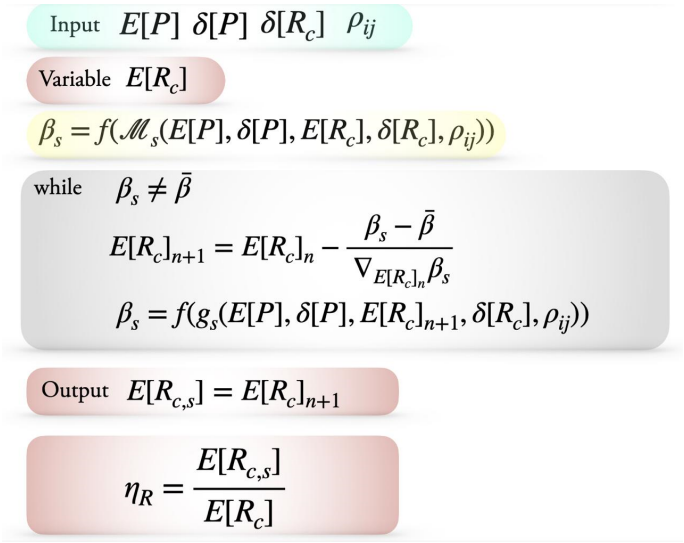

Figure 3: Procedure to assess the redundancy factor

Initially, the mean component resistance, $E[R_c]_1$, is determined based on the given target reliability, $\bar{\beta}$. The corresponding system reliability index, β_s, is then evaluated analytically or via (advanced) Monte Carlo Simulation (MCS)[3]. Typically, this value differs from the target reliability, $\bar{\beta}$. Next, the mean component resistance, $E[R_c]$, is iteratively updated, usually employing a Newton-Raphson scheme, until the system reliability satisfies the target value, i.e., $\beta_s = \bar{\beta}$ for a given $E[R_c]_s$. The redundancy factor is then computed as the ratio between $E[R_c]_s$ and $E[R_c]_1$.

1.3. Basic Examples on series, parallel and mixed systems

The initial phase of the project analyzed elementary systems using system reliability theory, as detailed in [1]. The objectives were to evaluate how system reliability depends on component configuration and probabilistic model parameters for loads and resistances, and to estimate the redundancy factor. At this stage, the system limit state function is directly formulated as a combination of component failures, without accounting for load redistribution based on the constitutive behavior (e.g., ductile vs brittle) of individual

[3]This summary report does not include the details of the importance sampling algorithms or the Gaussian Process-based surrogate models used later to estimate failure probabilities.

components. For brevity, only the most significant results are presented to support the observations in this section[4].

A key observation concerns the variation of the system reliability index, β_s, with the number of components. For series and parallel systems of n components, β_s was computed for resistance correlation coefficients ρ ranging from 0 to 1. The results, Figure 4, show that β_s decreases with increasing components in series systems, while it increases in parallel systems. However, the rate of change diminishes as the number of components grows. This suggests that subsequent analyses can be limited to a finite number of components, as variations in β_s become less significant. A second observation

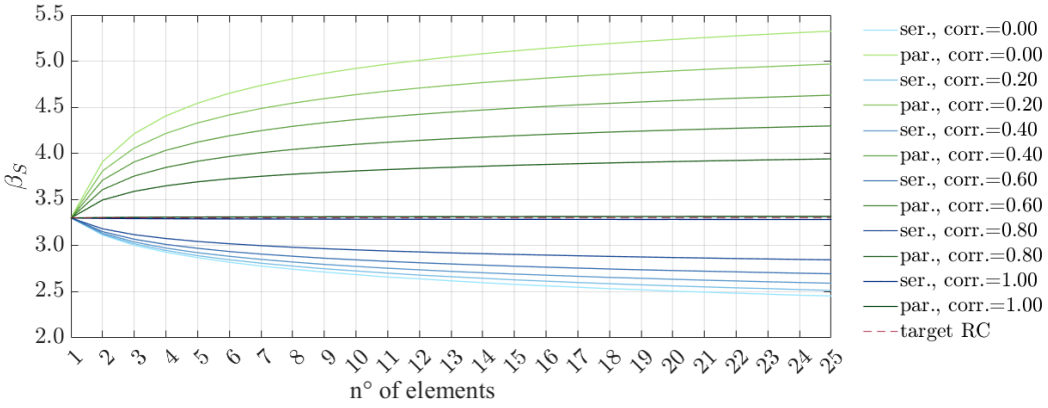

Figure 4: System reliability for series and parallel systems with increasing number of components and resistance correlation ranging between 0 and 1, for RC1 ($\beta_c = 3.3$) and gaussian distribution

is that resistance correlation positively impacts series systems by increasing system reliability, while it negatively affects parallel systems by decreasing system reliability. This indicates that the most critical condition arises in structures behaving as series systems with zero to low resistance correlation. Therefore, the case $\rho = 0$ for series system represents the lower bound for β_s.

The redundancy factor is analyzed for simple 3-component series, parallel, and mixed systems with a resistance correlation of $\rho = 0.5$. Figure 5 illustrates the system configurations, and the redundancy factor is computed following the procedure in Section 1.2. The results are presented in Figure 6, which shows system reliability curves as a function of η_R. The intersection

[4]More details are reported in the long version of this report

of these curves with the target reliability[5] determines the redundancy factor for each system, with the results summarized in Table 1.

As expected, $\eta_R > 1$ for series systems, indicating that component-level resistance must be increased, and $\eta_R < 1$ for parallel systems, reflecting a system safety reserve. As shown in Table 1, the redundancy factor values remain consistent across different reliability classes. For mixed systems, η_R is approximately 1 but varies depending on the ratio of parallel to series subsystems. Therefore, in series systems is necessary to increase the component

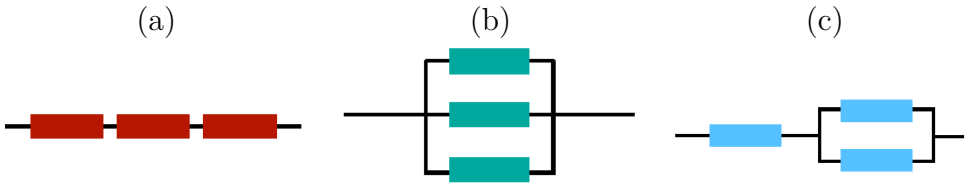

Figure 5: Representation of 3-component series (a), parallel (b) and mixed (c) systems

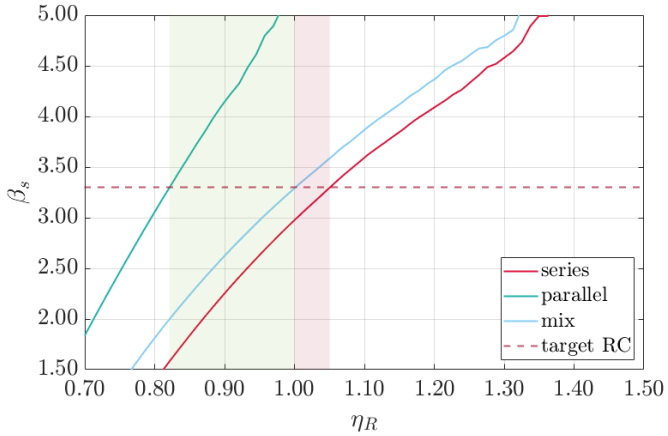

Figure 6: Redundancy factor for 3-component systems with resistance correlation $\rho = 0$, RC1 ($\beta_c = 3.3$), gaussian distribution

resistance, and consequently the component reliability index, to restore the target system reliability value. Conversely, for parallel systems there is a reserve of system reliability due to the parallel configuration of components

[5]RC1 with $\beta = 3.3$, RC2 with $\beta = 3.8$, and RC3 with $\beta = 4.3$

	RC1 $\beta = 3.3$	RC2 $\beta = 3.8$	RC3 $\beta = 4.3$
Series	1.051	1.047	1.043
Parallel	0.821	0.797	0.770
Mixed	1.001	0.999	1.002

Table 1: Redundancy factors for basic 3-component systems with $\rho = 0.0$

compared to the prescribed target value. It is also observed that, as the considered reliability class changes, the redundancy factor remains sufficiently stable.

System reliability and redundancy factor interaction was also analyzed in the presence of series groups within a more complex mixed system. For this purpose, a case study was modeled based on Figure 7. In this model, the deck beams are grouped as parallel components, while each prestressed tendon constitute a series component. The collapse of the entire structure is clearly govern by the series system constitutes by the red elements (tendons), which are identified as the critical components.

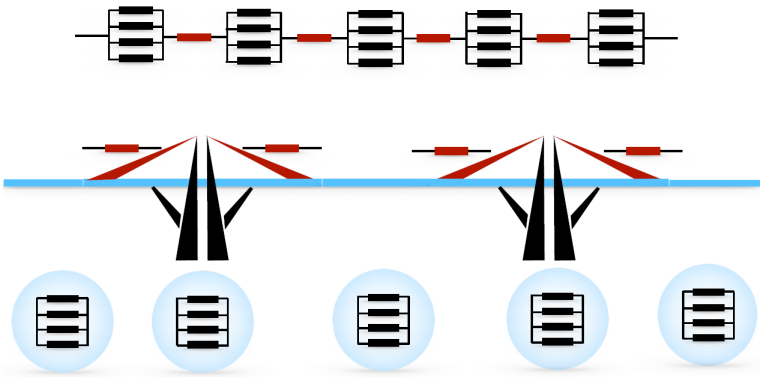

Figure 7: Schematization of a bridge structure as a combination of series (red) and parallel (blue) components

The assumptions regarding the distributions of loads and resistances were the same used for the analyses over 3-component systems. Resistance correlation ρ ranges from 0 to 1, and all the three reliability classes have been considered. The analyses were repeated for an increasing number of series components, computing for each case the redundancy factor η_R, similarly to what has been done for the 3-component systems.

For the sake brevity, only the results for RC1 with $\rho = 0.0$ are shown in Figure 8. The findings indicate that a structural system that includes even a single series component can reduce significantly the system reliability β_s. Specifically, with $n = 0$, $\eta_R < 1$ (so that $\beta_s > \bar{\beta}$). However, adding just one series component brings η_R to 1 (corresponding to $\beta_s = \bar{\beta}$). With two or more series components, $\eta_R > 1$ (for which $\beta_s < \bar{\beta}$). Therefore, it is possible to identify this subgroup of series components as "critical components." To restore system reliability β_s to the target value $\bar{\beta}$, it is necessary to act on these critical components by applying the redundancy factor.

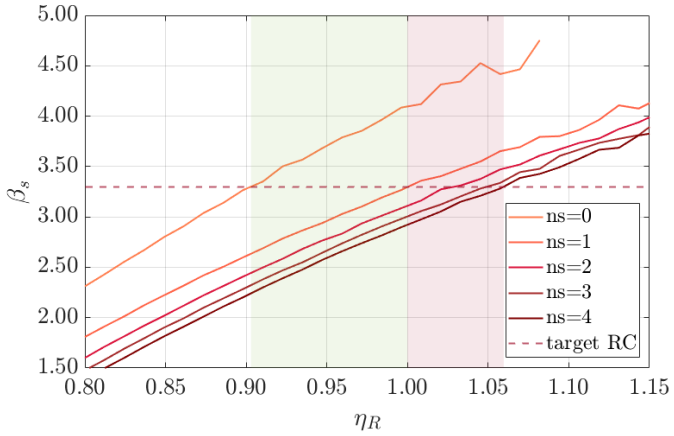

Figure 8: Redundancy factor for mixed systems with increasing number of critical components, resistance correlation $\rho = 0$, RC1 ($\beta_c = 3.3$), gaussian distribution

Key Observations

- System reliability is greater than the target value for structures that behaves more like parallel systems and less than the target value for series systems.
- Structures that behaves as series systems require intervention to restore system reliability, while parallel systems have a reserve of reliability that inherently provides greater system safety.
- The redundancy factor remains essentially stable across different reliability classes, meaning studies can focus on a single target value and extend conclusions to other RCs.
- Series and parallel systems represent extreme cases, defining boundaries for system reliability and the redundancy factor. It is useful to analyze intermediate cases in terms of component configurations to assess how the system behaves in such situations.
- In the case of a mixed system, it is possible to identify a subgroup of series components, referred to as "critical components," on which the redundancy factor should be applied to restore the system reliability.

1.4. System Reliability Approach for Two Typical Italian Bridges

The pure system reliability approach [1] was applied to case studies involving highway overpasses, specifically evaluating the system reliability of standard prestressed concrete bridges from the A31 Valdastico highway. This infrastructure includes numerous overpasses, all following two static schemes based on Gerber saddles: Type-1 with Gerber saddles in their canonical configuration and Type-2 with saddles located at the pier corbels. The deck configurations have two possible layouts, with either 3 or 4 prestressed concrete beams. Therefore, there are four bridge configurations that have been studied, combining each static scheme with each deck layout. These are shown in in Figure 9. At this stage, all components are assumed to have the same reliability, reflecting the ideal hypothesis of compliance with building code design. The objective was to estimate the redundancy factor η_R for these configurations.

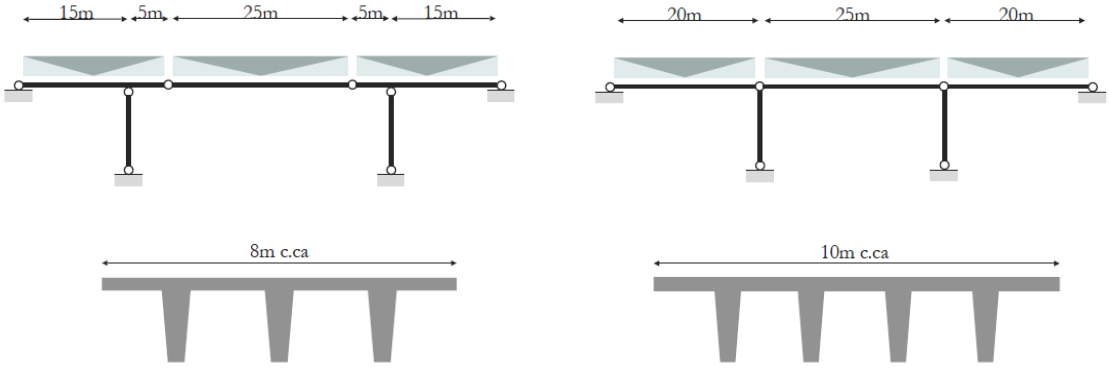

Figure 9: Type-1 (top-left) and Type-2 (top-right) standard static schemes, that have been combined with both 3-beams (bottom-left) and 4-beams (bottom-right) decks.

The components of the system are the Gerber saddles and the beams. Failures of the deck, columns, or supporting walls are not considered in this example, as their reliability is (here) assumed to be significantly higher than that of the aforementioned components. Moreover, the beams can fail either due to shear or bending moment. For each bridge configuration, four system failure limit state functions are defined by the following conditions:

A- Failure of any Gerber saddle or failure of either 1 external beam or 2 adjacent beams.
B- Failure of any Gerber saddle or failure of 2 adjacent beams.
C- Failure of any Gerber saddle or failure of 2 generic beams (not necessarely adjacent).
D- Failure of any Gerber saddle or failure of 1 generic beam.

Clearly, the hypotheses regarding failure modes lead to distinct values of the redundancy factor. Analyses were conducted to evaluate the variability of the redundancy factor, η_R, in relation to the structural failure hypotheses, with the correlation coefficient of component resistances varying between 0 and 1. For brevity, only the Type-1 four-beam deck bridge (shown in Figure 10) is presented, as it exhibited the greatest variability of η_R across different failure modes. Figures 11 shows the values of η_R for $\rho = 0$ (i.e., the scenario with higher values of η_R). Under these hypothesis $\eta_R \in [1.11, 1.15]$, indicating that this type of viaduct exhibits system reliability lower than the normative target. The other bridge configurations exibit η_R within this range.

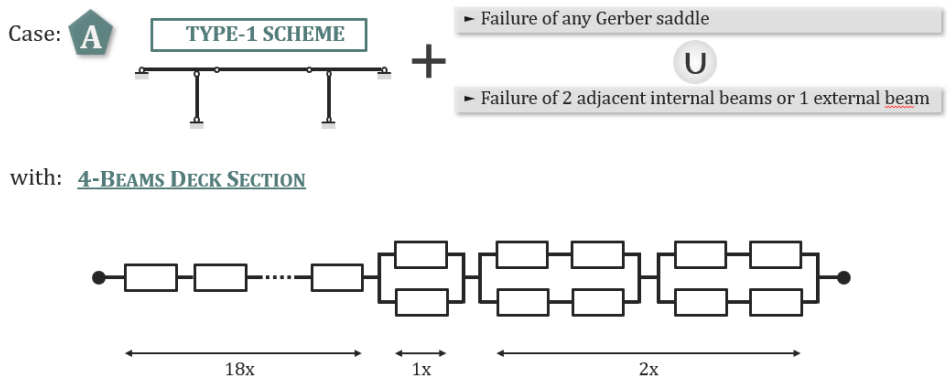

Figure 10: Block diagram for the type-1 standard bridge, according to point 1. hypotheses

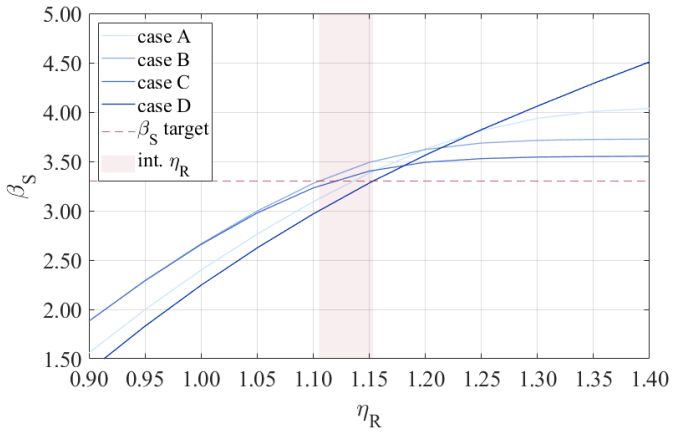

Figure 11: System reliability vs. redundancy factor for Type-1 scheme bridge with 4-beams deck, resistance correlation $\rho = 0$, RC1 ($\beta_c = 3.3$), gaussian distribution

In general, this approach has important limitations, as it does not account for the failure behavior of components (ductile or brittle) or the actual load-carrying capacity of the system. The assumptions about failure modes are predefined, and η_R is sensitive to these assumptions. As a result, this method provides only a range of possible values and is not strictly tied to real-world applications in structural mechanics. Therefore, it is deemed appropriate to introduce these aspects by transitioning from system reliability to structural system reliability.

Key Observations

- For local bridge systems, the redundancy factor ranges between 1.10 and 1.15, which is a relatively high value.
- The range of η_R is highly dependent on how the bridge is modeled, particularly on the assumptions made regarding the system collapse mechanism.
- This approach is not complete, as it does not consider aspects such as the presence of ductile/fragile members or load distribution among components.

1.5. Final notes and consideration

In this short version of the report, we do not provide a further in-depth analysis of critical subcomponents in mixed systems that govern overall system reliability. In such cases, each subgroup may exhibit different component reliabilities, allowing for the application of specific redundancy factors to critical components. However, we have chosen not to include this specific study for two reasons: first, it yields redundancy factors similar to those already discussed here; second, the analysis is superseded by the more comprehensive evaluations presented in Section 4. In case of interest, we refer the reader to the longer version of the report.

2. System Reliability and redundancy factor for Mechanical Systems

In this section, we compute the range of redundancy factors for systems where system reliability is governed by mechanical principles, considering appropriate load re-distributions. We analyze simple systems with constitutive models representing brittle and ductile behaviors. Despite their simplicity, these systems provide valuable insights into redundancy factor values.

2.1. System reliability and Redundancy factor for Daniels Systems

The Daniels system is a statistical model widely used in the literature (as in [3] [4], and [5] to study structural system reliability in the presence of load redistribution. In this model, a set of structural components, each with a random strength, is subjected to a gradually increasing load, as depicted

in Figure 12. The load is redistributed according to the stiffness of each structural component. The system is considered to have failed when the applied load exceeds the total remaining strength of the surviving elements, i.e., when static equilibrium can no longer be maintained. The response of

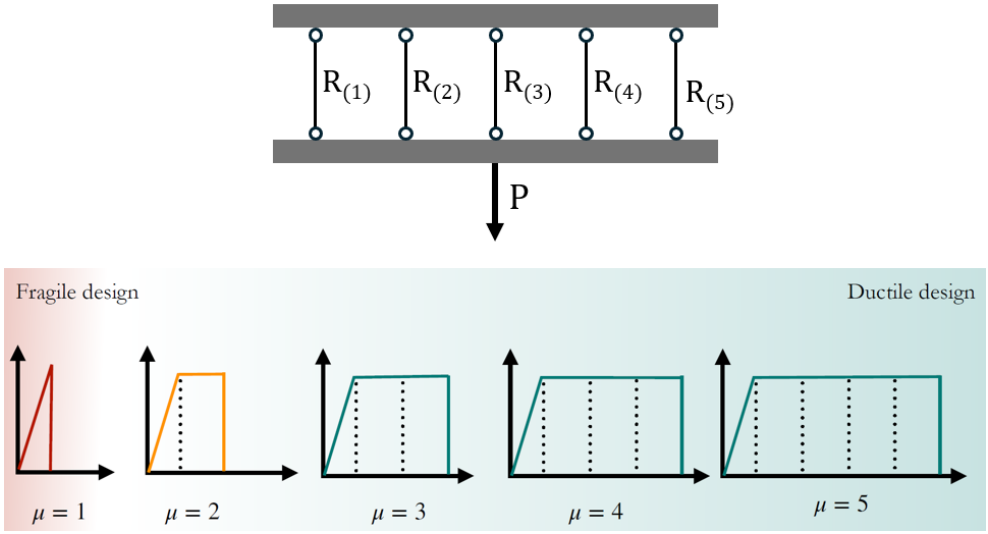

Figure 12: Sketch representation of a 5-component Daniels System (top), with load/displacement functions for its components (bottom) for increasing values of ductility μ

the Daniels system in terms of system reliability is significantly influenced by the constitutive models of its components. To account for this aspect, we establish the mechanical behavior of each component through a linear elastic-perfectly plastic load-displacement relationship. To characterize the failure behavior, we introduce the ductility coefficient, μ, defined as the ratio between the ultimate strain and the yield displacement:

$$\mu = \frac{\delta_u}{\delta_y}. \tag{7}$$

Therefore, for $\mu = 1$, the failure is brittle, while increasing μ results in more ductile behavior. Specifically, $\mu = \infty$ represents the ideal case of elastic-perfectly plastic behavior. For reference, see again Figure 12.

An equal, constant stiffness K is assumed for all components, while each component's load is considered a random variable, as previously described. The Daniels system is then studied under three scenarios: all components

are brittle (brittle system), all are ductile (ductile system), or a combination of brittle and ductile components (mixed system). All other assumptions regarding component loads, resistances, and probabilistic models remain consistent with those presented in Section 1.2. Thus, the Daniels system introduces hypotheses related specifically to components' mechanical behavior and load distribution.

2.1.1. System reliability for Daniels systems

Daniels System capacity and reliability can be computed from the given resistances of each component. The formulation depends on the components failure behavior: whether they are ductile, brittle, or a combination of the two. Here we present first the three ideal cases where: i- $\mu = 1$ for all components (brittle system, whose load/displacement is shown in Figure 13 (a)); ii- $\mu = \infty$ for all components (ideal elasto-plastic system, whose load/displacement behavior is shown in Figure 13 (b)); iii- a system with both brittle components and ideal elasto-plastic components (see Figure 13 (c) for its load-displacement behavior). Finally, we present the general case where the components have finite ductility.

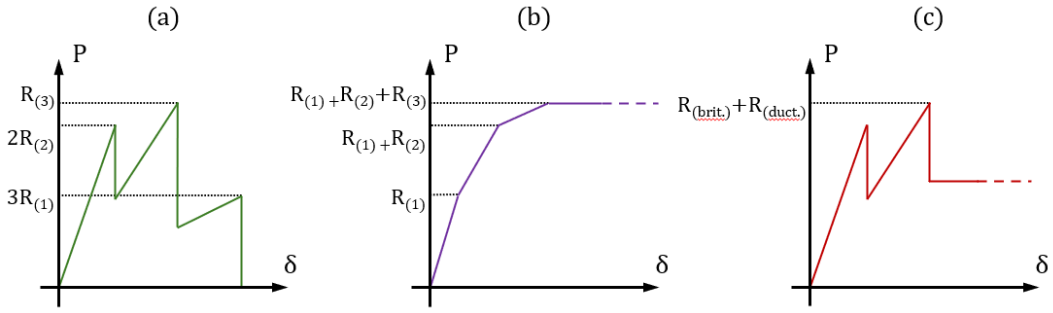

Figure 13: Load/displacement functions for Daniels System with brittle-only components (a), with only ideal elasto-plastic components (b), and with a combination of the two (c)

i- **Daniels system with all brittle components**. Given n-components with ductility $\mu = 1$, the system capacity can be computed as:

$$\begin{aligned} R_S &= \max[nR_1;\ (n-1)R_2;\ (n-3)R_3;\ ...;\ R_n] \\ &= \max_{j=0}^{n-1}[(n-j)R_{j+1}], \end{aligned} \tag{8}$$

where $R_1 < R_2 < R_3 < ... < R_n$ are the j-components resistances sorted from lower to higher. Despite there is no close form solution for the probability distribution of R_S, there are asymptotic solutions presented in [4]. Moreover, provided with Eq. (8) the computation of the empirical distribution is straightforward and essentially costless.

ii- **Daniels system with infinity ductility**. Given n-components with ductility $\mu = \infty$, the system capacity can be computed as the sum of all components resistances:

$$R_S = \sum_{j=1}^{n} R_j. \tag{9}$$

When component resistances in a Daniels system follow a Gaussian distribution, R_S is Gaussian too. In other cases, empirical solutions are straightforward and often preferred over asymptotic solutions based on the central limit theorem. This solution works well also if all components exhibit large values of ductility.

iii- **Daniels system with both ductile and brittle components**. Given n-components with $\mu = 1$ and m-components with $\mu = \infty$, it can be shown:

$$R_S = R_{brit.} + R_{duct.} \tag{10}$$

Where $R_{brit.}$ stands for the resistance of the brittle components, while $R_{duct.}$ stands for the resistance of the ductile components. Both can be computed as shown on previous points. Note that, to reach the maximum system capacity, all ductile components must fail.

The more general case is represented by a Daniels system composed of components characterized by finite ductility. To assess the system's capacity in this configuration, it is necessary to perform a pushdown analysis using a displacement-controlled algorithm. As previously mentioned, each component's behavior is described by a bilinear load-displacement function. Given a certain system displacement, δ (which, for the Daniels system, is uniform across all members), the status of each component can be identified as elastic, plastic, or failed, depending on its capacity:

$$P_{comp.} = \begin{cases} K\delta & \text{if } \ 0 \leq \delta < \delta_E \\ P_y & \text{if } \ \delta_E \leq \delta < \delta_u \\ 0 & \text{if } \ \delta \geq \delta_u. \end{cases} \tag{11}$$

At each iteration, the system's capacity is computed as the sum of the capacities of all components. By applying this procedure to a set of increasing displacements, $\overline{\delta}$, the system's load-displacement curve can be constructed, allowing for the determination of its maximum capacity. This method is applicable to systems comprising ductile, brittle, or mixed components.

Several analyses have been performed over different Daniels Systems, according to the assumptions introduced in Section 1.1 and Section 1.2. For the sake of brevity, only the main results will be described within this section, while more extensive results are presented in the extended report. For the components load and resistance probability models, both normal and Log-normal distributions have been considered, but here we report only the latter case.

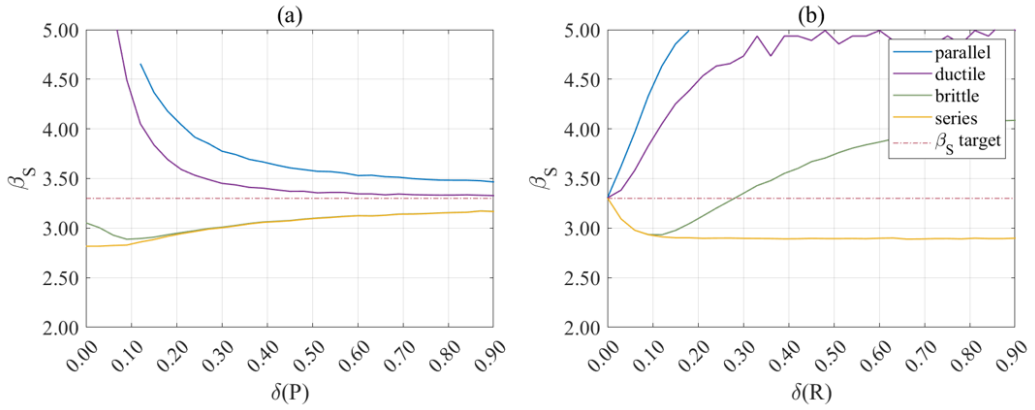

Figure 14: Daniels system with 5 components. The correlation between resistances is fixed to $\rho = 0$, which represents the worse condition for the system reliability. a) System reliability index β_s as function of load $\delta[P]$ and fixed resistance $\delta[R] = 0.1$. b) System reliability index β_s as function of load $\delta[R]$ and fixed resistance $\delta[P] = 0.1$.

Figure 14 (a) illustrates the system reliability index as a function of load variability, with a fixed $\delta[R] = 0.1$. Conversely, Figure 14 (b) also presents the system reliability index as a function of resistance variability, with a fixed $\delta[P] = 0.1$. The figure reports results for two ideal cases: a Daniels system composed entirely of brittle elements (brittle system) and a system with ideal elasto-plastic structural members (referred to as the ductile system). Additionally, the figure includes the ideal parallel and series cases without load redistribution.

Several conclusions can be drawn. First, as load variability increases, brittle systems exhibit behavior similar to series systems. This occurs because when one component fails, the entire load is transferred to the remaining components, which lack sufficient capacity, resulting in cascading failure. Second, the parallel system represents an upper bound for ductile systems, which consistently show lower reliability. This is due to the fact that, in an ideal parallel system, each component must be designed to sustain the full applied load, P. An interesting observation can be made from Figure 14 (b): as the variability of R increases, brittle systems exhibit a non-monotonic behavior. Specifically, system reliability initially decreases, reaching a minimum, and then starts to increase. This behavior arises from the fact that higher variability in R increases the probability of both weak and strong components. Consequently, weaker components redistribute less load, which can be absorbed by the presence of stronger ones. Furthermore, brittle systems behave like series systems for low $\delta[R]$ values and significantly diverge for higher values.

Figure 15 depicts the system reliability as a function of the number of components. The observed trend mirrors that of Figure 4. In this scenario, the brittle system approximates a series system, serving as a lower bound. Conversely, the ductile system exhibits a trend similar to that of a parallel system, though with notable differences in magnitude.

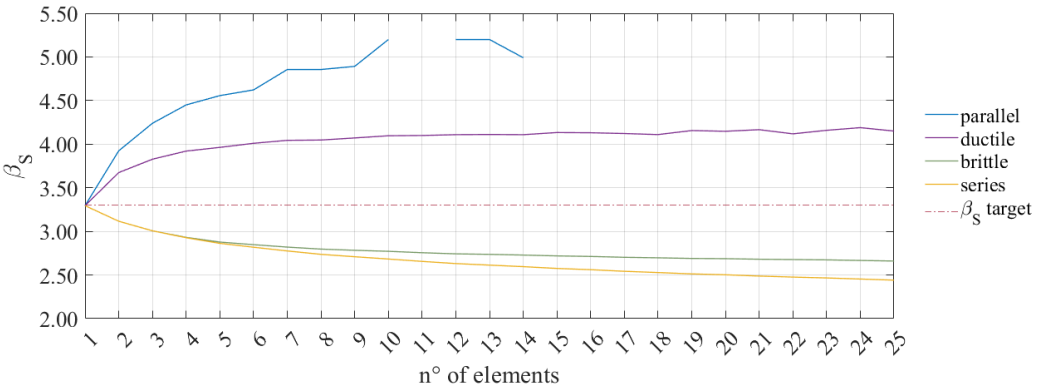

Figure 15: System reliability index β_s of Daniels Systems with increasing number of components, assuming resistance correlation $\rho = 0$, RC1 ($\beta_c = 3.3$) and lognormal distribution

2.1.2. Redundancy factor for Daniels systems

The redundancy factor, η_R, was evaluated using the procedure outlined in Section 1.2, with the system's loss of static equilibrium considered as the limit state function. Analyses were performed by varying the resistance correlation coefficient, ρ, the number of components, and their failure behaviors as previously described. Both Gaussian and Lognormal probability models were examined. In this summary report, Figure 16 presents the results for a Daniels system consisting of five components, considering brittle, ductile, and mixed configurations (i.e., combinations of brittle and ductile components). The probabilistic models assume Lognormal distributions with $\delta[R] = \delta[P] = 0.1$, and the resistance correlation is fixed at $\rho = 0$, representing the most unfavorable condition for brittle systems. The first reliability class is considered, with similar results observed across different classes. The results indicate that the brittle system exhibits a redundancy factor close to that of an ideal series system, while the ductile system shows redundancy factors significantly higher than those of a parallel system. Interestingly, in mixed systems, a relatively small fraction of ductile members is sufficient to achieve $\mu_R \leq 1$.

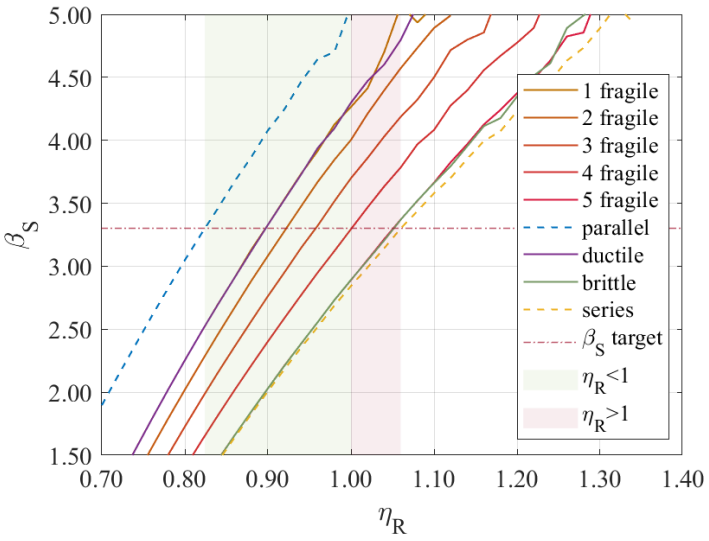

Figure 16: Redundancy factor for ideally-parallel, ideally-series, brittle and ductile Daniels systems with 5 components. Load and resistance distributions are lognormal, with fixed resistance correlation $\rho = 0$ and for RC1 ($\beta_c = 3.3$)

Figure 17 presents the system response for ductility factors, μ, ranging from 1 (representing ideally brittle behavior) to 5 (representing a highly ductile behavior). The results indicate that even with a relatively low ductility coefficient of $\mu = 2$, the system behaves similarly to one composed of ideally ductile components. This underscores the significant influence of component ductility on the overall system response. Ductile components exhibit system behavior similar (in trend) to an ideal parallel system, providing a substantial reserve of system reliability—albeit significantly lower than that of an ideal parallel system. This occurs because when a ductile component yields, it retains its load-bearing capacity, allowing the remaining components to redistribute only the incremental load increases. Conversely, as previously mentioned, when a component exhibits brittle behavior, it fails completely, losing its load-bearing capacity. Consequently, the remaining components must carry not only the additional load but also the portion previously borne by the failed component. This greatly increases the likelihood of cascading failures, making a brittle system behave (very) similarly to a series system. It is important to report than in our full analysis we discovered narrow range of redundancy factors for brittle systems, typically of the order of $\eta_R \leq 1.1$.

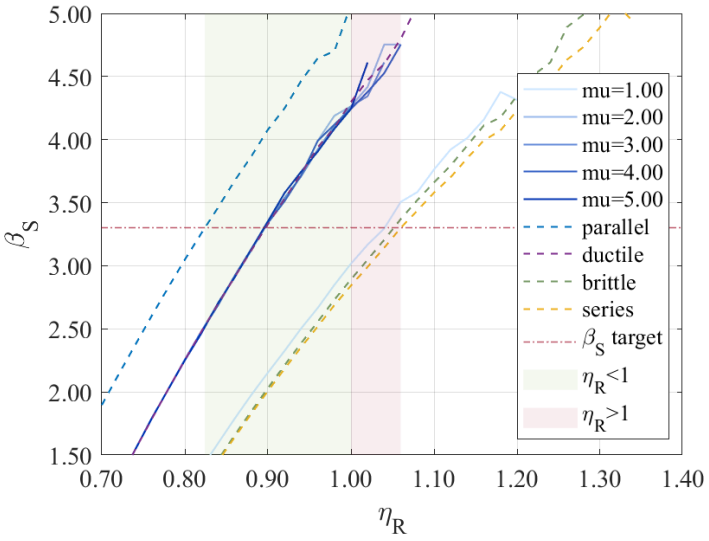

Figure 17: Redundancy factor for Daniels systems of 5 components and increasing ductility μ. Load and resistance distributions are lognormal, with fixed resistance correlation $\rho = 0$, and RC1 ($\beta_c = 3.3$)

Key Observations

- Ideally parallel and ideally series systems serve as boundaries for "real" systems, where structural mechanics have been incorporated.

- Systems with only brittle components behave similarly to series systems, with the series system representing a close lower bound. This occurs because the failure of a single element can lead to a cascading failure, ultimately resulting in the failure of the entire system. Conversely, systems composed solely of ductile components behave similarly to parallel systems, though the upper bound is significantly higher. This is because an ideal parallel system assumes that each component is designed to carry the full load independently of the others.

- A relatively low ductility coefficient ($\mu = 2$) is sufficient to transition from brittle-like to ductile-like behavior.

- The presence of a fraction of ductile elements can significantly enhance system reliability and consequently lower the redundancy factor. Conversely, the presence of brittle components decreases system reliability and increase the redundancy factor.

- The redundancy factor values for the examined brittle cases are typically $\eta_R < 1.1$.

3. System Reliability and redundancy factor for Courbon like model of existing bridges

The Daniels System introduces the failure behavior of components into the system reliability problem, as well as the system's ability to redistribute load or accommodate subsequent load increments. Another fundamental aspect to consider when transitioning from pure system reliability to structural system reliability is the distribution of applied loads among the various components, depending on their stiffness and position. This is well exemplified in the case of a bridge deck subjected to a eccentric load.

3.1. Reliability on Courbon-modeled bridges

A model of a typical deck was thus introduced, defined according the classical Courbon theory (i.e., a perfectly rigid deck, Figure 18). The static scheme is a simple supported span. In this analysis an equivalent load[6] is applied at mid-span, leading to beam failure due to the either the maximum resisting moment or the maximum shear being reached. The failure of other elements, such as pillars, retaining walls, or the distribution slab, is not considered in this analysis. The load eccentricity varies between 0 to 3 m. Other assump-

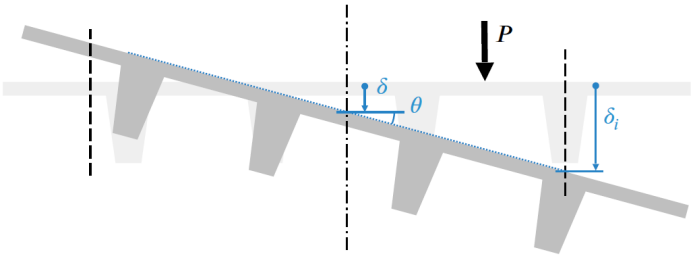

Figure 18: Schematic representation of a typical bridge deck according to Courbon model

tions include the Lognormal distribution of loads and component resistances, similar to what was described in Section 1.2. The following assumptions regarding component design were made:

- **Equal Reliability Design (ERD)**. (Figure 19 (a)). Each component is designed and optimized based on its maximum applied load. As a result, beams have different sections and, consequently, varying resistances, with edge beams having higher resistance. However, all components achieve the same reliability level. Although this design approach is not commonly used, it complies with regulatory guidelines that allow for the optimization of individual components, making it relevant for inclusion in this study.

- **Equal Component Design (ECD)** (Figure 19 (b)). The most stressed component, typically the edge beam, is designed and optimized, and the resulting section is applied to all other elements. This approach results in identical resistance across all beams, leading to overdesign of

[6]i.e., a load that produce the same effects of a given load combination

the inner beams and, consequently, higher reliability. This method has been the most commonly adopted practice in the design of viaducts and highway overpasses in Italy.

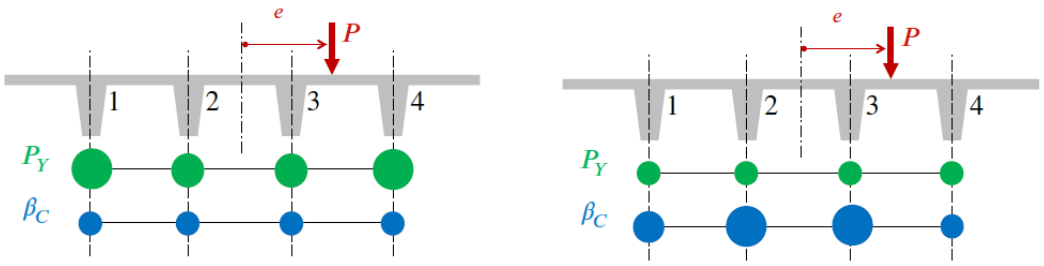

Figure 19: Design load and reliability index for beams designed according to equal reliability (left, (a)) and equal component (right, (b)) criteria

As in previous analyses, the study was conducted for Gaussian an Log-normal distributions with $\delta[R] = \delta[P] = 0.1$ and different resistance correlation values, ranging from 0 to 1. The beams were analyzed considering both brittle behavior, modeled with a linear elastic relationship, and ductile behavior, represented by a linear elastic-perfectly plastic relationship. The load-deformation relationships were bilinear, with a uniform stiffness K for all elements. The maximum resistance was treated as a random variable, while ductility was fixed at $\mu = 1$ for brittle elements and $\mu > 2$ for ductile elements.

To determine the maximum load capacity of the deck for a given eccentricity, a displacement-controlled Push-Down analysis is performed. At each iteration, given the displacement, the condition of the components (surviving, yielding, or failed) is evaluated, and the reaction of the deck is assessed.

3.1.1. The redundancy factor for Courbon like model of existing bridges

In this summary report, we present the results for a four-beam Courbon-like simply supported bridge with a resistance correlation of $\rho = 0$ and considering the first reliability class (with similar results observed for higher reliability classes).

Figure 20 (a) illustrates the redundancy factor for the ERD configuration with brittle components, while Figure 20 (b) presents the redundancy factor

for the ERD configuration with ductile components. In the first case, the redundancy factor $\eta_R > 1$ for all eccentricities. The brittle failure behavior of the components results in significant overloads on the remaining elements following the first failure. However, since all components are optimized with no additional resistance margins to absorb such overloads, a cascading failure occurs, leading the system to behave similarly to a series system—where the failure of any component results in system failure. In the second case, the redundancy factor is lower and $\eta_R > 1$ only for the maximum eccentricity. Yielding in the components prevents a complete loss of resistance, ensuring that even without significant resistance margins, the remaining components only need to accommodate subsequent load increments.

Notably, system reliability improves with increasing eccentricity in the case of brittle components. Conversely, for ductile components, an increase in eccentricity leads to a decrease in system reliability. This trend suggests that higher eccentricity causes the system to behave more like a two-component system (failure of one of the external beams), which becomes less sensitive to the ductile or brittle nature of the components. When eccentricity is zero, the average loads and resistances are equal across all components, making the system behave like a Daniels system.

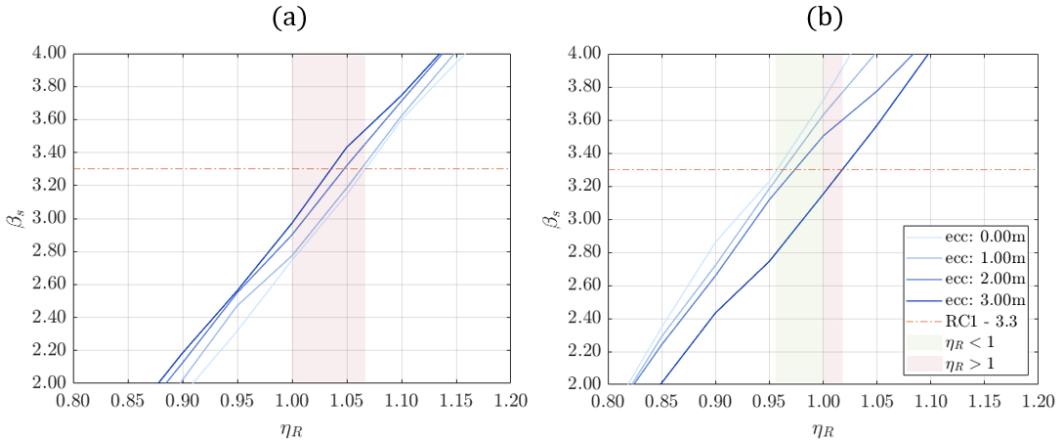

Figure 20: Redundancy factors of a 4-beams Courbon deck-type bridge with brittle (a) and ductile (b) components. Bridge is designed with ERD, the correlation between resistances is fixed to $\rho = 0$, reliability class is RC1 ($\beta_C = 3.3$)

Figure 21 (a) illustrates the redundancy factor for the ECD configuration with brittle components, while Figure 20 (b) presents the redundancy fac-

tor for the ECD configuration with ductile components. Both with brittle and ductile elements, the redundancy factor is lower compared to the ERD. This is due to the presence of over dimensioned elements capable of absorbing significant load increments. In this case, the presence of eccentricity is advantageous: the edge beams are considerably more stressed, resulting in a greater reserve of strength for the inner beams. The effect of the failure behavior of the components (brittle/ductile) is similar to what was observed with the ERD.

The redundancy factor, η_R, falls within a considerably narrower range compared to the typological examples analyzed in Section 1.4. The observed η_R values are, in fact, closer to those obtained for the Daniels systems. However, caution must be exercised when drawing direct comparisons, as the mechanical system analyzed here is highly simplified. Nevertheless, it offers a significant advantage by providing an (almost) standardized framework for redundancy factor calibration, which effectively accounts for load redistribution effects.

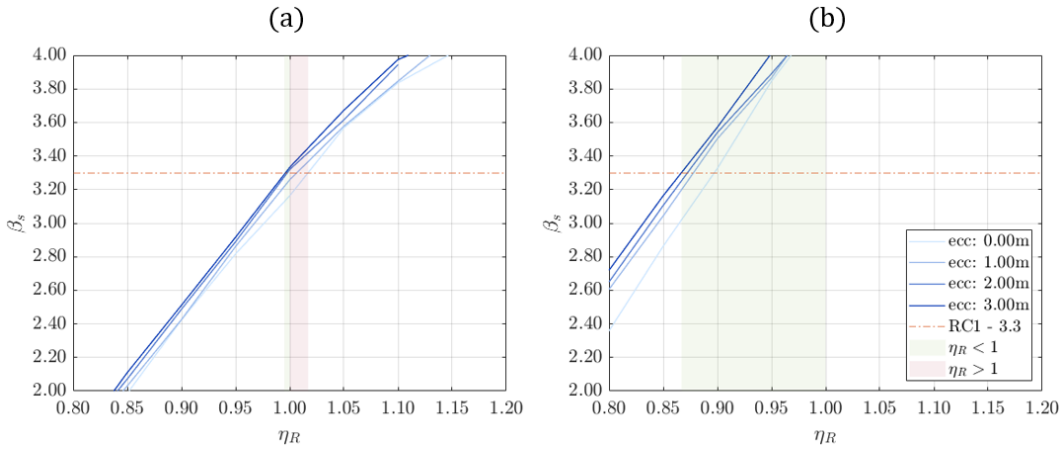

Figure 21: Redundancy factors of a 4-beams Courbon deck-type bridge with brittle (a) and ductile (b) components. Bridge is designed with ECD, the correlation between resistances is fixed to $\rho = 0$, reliability class is RC1 ($\beta_C = 3.3$)

Key Observations

- We investigate two bridge design configurations for Courbon-like simply supported bridges: Equal Reliability Design (ERD) and Equal Component Design (ECD).
- For ERD with brittle components, the redundancy factor is consistently within the range $\eta_R \in [1, 1.1)$. For ERD with ductile components, η_R is generally less than 1, except for high eccentricity cases where it falls within $\eta_R \in [1, 1.1]$.
- For ECD with brittle components, the redundancy factor is always within $\eta_R \in [1, 1.1)$ but exhibits lower values compared to ERD. For ECD with ductile components, η_R is generally less than 1. In both cases, the lower values of η_R are attributed to the overdesign of the inner beams.

Conclusion and Final Remarks

The NTC 2018 and Eurocodes do not explicitly address system-level reliability targets, posing a critical gap, particularly for bridge infrastructure. This study, inspired by the AASHTO approach, investigates the introduction of a resistance-level partial safety factor, named the redundancy factor, η_R , to align component and system reliability targets. We propose the application of η_R at component resistance level of critical brittle component failures.

We examined the calibration approach proposed by [1] and incorporated a structural system reliability approach for simplified mechanical models representative of common bridges. The findings indicate that for systems with brittle failure-prone elements, η_R is a recommendable partial safety factor to prevent cascading failures and potential system collapse.

The study aims to provide insights into redundancy factor values using simplified and standardized mechanical models. It is important to note that the proposed calibration is independent of conventional component-level partial safety factor calibrations. For systems exhibiting brittle failure behavior, the redundancy factor ranges within $\eta_R \in [1, 1.1]$. These results provide a theoretical basis for further research and potential integration into Italian and European design standards.

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